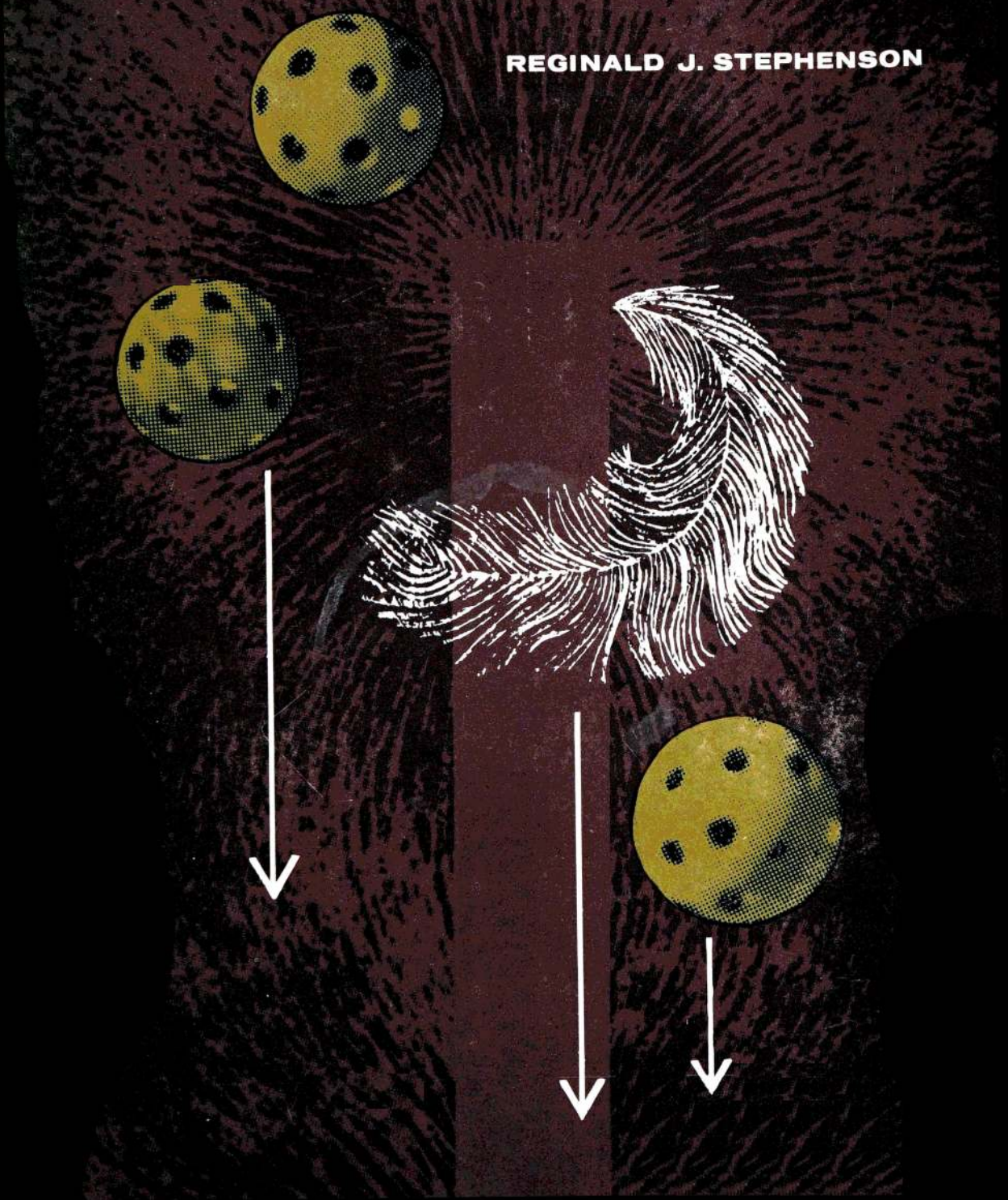
PHOENIX SCIENCE SERIES
PSS 505 $1.50

exploring in physics

REGINALD J. STEPHENSON



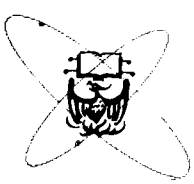
EXPLORING IN PHYSICS

EXPLORING IN PHYSICS

A NEW OUTLOOK ON PROBLEMS IN PHYSICS

BY

REGINALD J. STEPHENSON



PHOENIX SCIENCE SERIES

THE UNIVERSITY OF CHICAGO PRESS

PREFACE TO THE READER

THE SCIENCE of physics has advanced as far as it has partly because its followers have found it so fascinating. You will be able to discover some of this fascination for yourself if you will but try.

When you learn some law, remember it is a grand generalization which often came to the mind of a genius after consideration of a lot of special cases. Try and think of some of the special cases to which the law applies and you will find a whole new world opened up to you. These cases are not so hard to discover in your everyday experience. The discussion material, the examples, problems, and questions—in fact, the whole book—is an attempt to help you to see this interesting new world. Go out as an explorer would and see what you can find! You must, however, be willing to spend some time and effort in this exploration, for there is no royal road to knowledge.

PREFACE TO THE TEACHER

THE NATURE of this exploration into the field of physics is not so much that of an investigation of new phenomena as a relooking at some of the old, well-established ones.

When Professor Harvey B. Lemon had published his volume, *From Galileo to Cosmic Rays*, later revised and renamed *From Galileo to the Nuclear Age*, it was felt that a smaller volume, comprising examples and problems, should accompany it. This book, which Professor Lemon has given me the privilege of writing, may be considered to be a direct offspring of his book.

WE HAVE felt that for elementary students physical principles are often blurred and confused by the mathematical difficulties encountered when both the metric and the British system of units are used. The British system of units has therefore been dropped except in a few instances. Once a student has mastered the concepts using the metric system, it is a small matter to transfer to the British system.

Each chapter has a few pages of discussion introducing the physical laws or principles used in the problems. Examples and questions are scattered throughout this discussion so that the student may be able to learn the methods of attacking the problems.

A somewhat new approach has been made in many of the problems in that the problem has been so subdivided that the answer to one part may be suggested by the preceding part. Thus the student is able to attack more complex problems than would otherwise be possible.

A number of non-numerical questions have been included as it is thought

that these can often bring a concept more vividly and clearly to a student's mind than can a numerical problem.

In the section on electricity the units may almost seem to have been treated with disrespect, yet, we hope, without any loss in rigor in the presentation of the principles themselves. The several systems of units used in electricity present a difficult problem for the elementary student. It was felt that most of the fundamental principles could be more readily understood if only the practical system of units were used.

Faraday's Laws of Electrolysis have been related to the periodic table, thus eliminating the necessity of a previous knowledge of chemistry and chemical valence on the part of the student.

Wave motion is discussed in considerable detail because we feel that this subject possesses unusual difficulties for students. Such a general chapter on wave motion enables the chapters on light and sound to be brief.

A final chapter on waves and particles has been added more to show the student the dilemma with which physics has been faced rather than to attempt the impossible of presenting to beginners an explanation.

A short Appendix giving a few of the simple mathematical methods used in the text has been added.

Finally, it should be said that some of the material usual to this type of book has been omitted on the grounds that less material well understood is far better than a mass of material presented for completeness but understood little or not at all. If this book helps a student to think about physical principles and to derive pleasure from it, then the author will feel he has succeeded in his main objective.

WE WISH to offer our sincere thanks to Professor Harvey B. Lemon, who made this book possible and who has given valuable help in its preparation, as well as permission to use many illustrations from his book, *From Galileo to the Nuclear Age*. We are also grateful for the help and criticism liberally given by Mrs. Ardis T. Monk, a fellow-member of the physical science staff. Grateful acknowledgment is made to Dr. John H. Clements, who has checked the problems and examples; to Chichi Lasley, whose drawings have done much to make graphic some of the physical concepts; and to Robert A. Millikan, for the use of figures on pages 186 and 190 from his *Electrons (+ and −), Protons, Photons, Neutrons, and Cosmic Rays.*

REGINALD J. STEPHENSON

RYERSON PHYSICAL LABORATORY
UNIVERSITY OF CHICAGO
August 16, 1935

CONTENTS

1. Speed and Acceleration 1
   Units 1
   Speed—Uniform, Average, Instantaneous 4
   Acceleration 5
   Examples 7
   Problems 9

2. Newton's First Two Laws of Motion 13
   Inertia . . . 13
   Mass 14
   Newton's First Two Laws: Force—Dyne 14
   Acceleration—Linear, Circular . 15
   Weight 16
   Newton's Law of Gravitation . 17
   Examples 18
   Problems . . . 20

3. Addition of Vectors. Projectiles . 25
   Composition and Resolution of Vectors 26
   Acceleration of Object on a Smooth Inclined Plane 28
   Composition of Velocities . 29
   Projectiles 30
   Problems . . . 32

4. Newton's Third Law of Motion . 37
   Statement and Discussion of Law . . . 37
   Law of Conservation of Momentum at Impact . 40
   Collisions—Elastic and Inelastic . . 41
   Examples . 43
   Problems . 44

5. Work and Energy 47
   Definition of Work 47
   Work against Frictional Forces . 48

Work against Gravity 49
Work against Inertia 50
Kinetic Energy 52
Potential Energy 53
Principle of Conservation of Mechanical Energy . . . 53
General Principle of Conservation of Energy 56
Levers 56
Definition of Moment of a Force 57
Determination of Mechanical Equivalent of Heat with Friction Cones . 58
Examples 59
Problems 59

6. Atmosphere and Hydrosphere 67
Definition of a Fluid . . . 67
Pressure 68
Transmission of Pressure . . 70
Mercury Barometer . . . 71
Boyle's Law 72
Archimedes' Principle . . . 73
Examples 74
Problems 76

7. Temperature and Expansion . . 80
Temperature 80
Comparison of Fahrenheit and Centigrade Scales 81
Expansion of Solids 81
Expansion of Liquids . . 82
Expansion of Gases 82
Volume Expansion of a Gas Heated at Constant Pressure 83
Charles's Law 84
Absolute Temperature . . 84
General Gas Law 84
Pressure Increase of a Gas Heated at Constant Volume 85
Problems 86

8. Heat 90
Definition of the Calorie . 91
Definition of Specific Heat 91
Measurement of Specific Heat by the Method of Mixtures 91
Change of State . . . 92
Latent Heat of Fusion 92
Measurement of Latent Heat of Fusion of Ice . . 93
Latent Heat of Evaporation 93
Measurement of Latent Heat of Evaporation of Water 94
Conduction, Convection, Radiation . 94
Problems 96

9. Molecular and Kinetic Theory of Matter . . . 99
Definition of the Atom and Molecule 99
Avogadro's Hypothesis 100
Molecular Weight 100
Kinetic Theory of Gases 101

Interpretation of Boyle's Law, Charles's Law, and Avogadro's Hypothesis . 102
Vapor Pressure 103
Saturated and Unsaturated Vapors 104
Maximum Vapor-Pressure Curve for Water . . . 104
Relative Humidity 105
Problems 105

10. Electric Charges and Magnetic Poles . . . 108
Coulomb's Law of Force . . . 108
Electrostatic Induction 109
Electric-Field Strength 110
Magnetic Fields 111
Work against an Electric Field . . . 113
Potential Difference 114
Electric Currents 115
Ohm's Law 116
Series and Parallel Circuits 117
Specific Resistance 119
Magnetic Field Produced by a Current 120
Force Exerted on a Wire Carrying a Current in a Magnetic Field . . . 121
Electromagnetic Induction 121
Problems 123

11. Conduction of Electricity in Liquids and Gases . . . 129
Rutherford-Bohr Atom 129
Periodic Table 131
Faraday's Laws of Electrolysis 133
Conductivity of Gases 134
Determination of the Mass of an Electron . . . 135
Positive Ions 137
Radioactivity 137
Artificial Disintegration 139
Problems 140

12. Wave Motion 143
Transverse and Longitudinal Waves . . 144
Velocity of a Wave 146
Wave Fronts and Rays 147
Reflection and Refraction of Waves . . 147
Superposition of Waves . . 149
Diffraction 151
Doppler Effect . . 152
Polarization . . 153
Problems . . 154

13. Sound . . . 156
Resonance, etc. . 157
Beats . . . 159
Problems . 160

14. Light 166
Mirrors 167
Refraction of Light Waves 170

Lenses 171
Interference in Light 174
Diffraction of Light 175
Diffraction Gratings 176
Problems 177

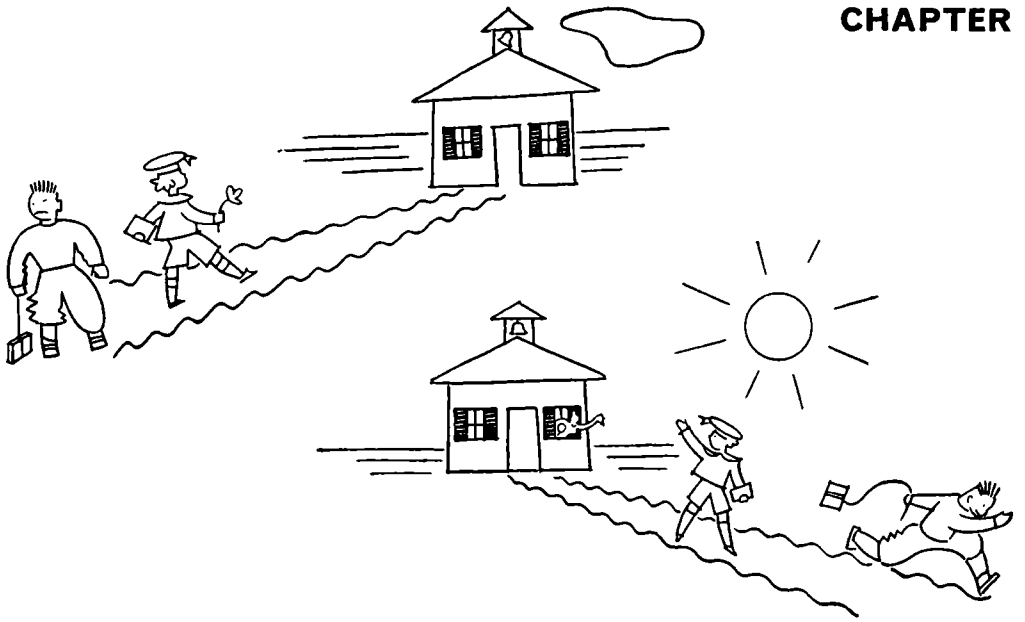
15. Electromagnetic Radiations: Waves and Particles . . 182
Continuous Spectra 183
Stefan's and Wien's Laws 183
Spectrum of the Hydrogen Atom 186
X-rays 188
Particle Characteristics of Waves 189
Wave Characteristics of Electrons . . . 190
Problems 192

Appendix . . . 194

Answers to Problems 197

Index 203

CHAPTER 1



SPEED AND LINEAR ACCELERATION

When you cannot measure, your knowledge is meager and unsatisfactory.—LORD KELVIN.

EXACT MEASUREMENTS are the makings of a science. In order to make measurements a system of units or standards must be adopted. If you wish to say how tall you are you might say that you are 5 ft. 9 in., or whatever your height is. The feet and inches are units of length and, as you probably already know, there is a definite relationship between the foot and the inch, namely, 1 ft. = 12 in.

You may know the distance of your home from school, but probably far more exactly the time necessary to go from home to school to make a nine o'clock class. It may take you 11 min. to get to school, which is $\frac{3}{4}$ mi. away, 1,320 yd. (1 mi. = 1,760 yd.). There is, of course, no relationship whatever between units of distance or length and units of time. They are such completely different concepts that you would never mix them up.

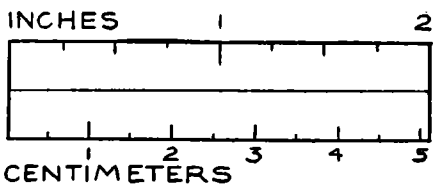
Your speed is 1,320 yd. in 11 min. or $\frac{1{,}320}{11}$ yd. per min., i.e., 120 yd. per min. SPEED then involves no new or separate units, but uses the two already mentioned, length and time. Length and time are called fundamental units. In the discussion of mechanics it is necessary to use only three fundamental units: LENGTH, TIME, AND MASS. The latter will not be used until Newton's laws are encountered, so we shall not discuss it at present.

UNFORTUNATELY for us there are two systems of fundamental units in common use, namely, the BRITISH and the METRIC systems. The British system of units is so awkward to use compared to the Metric system that it is regrettable that it is used at all—maybe some day we shall see this altered and the Metric system used as much in everyday practice as it is in scientific circles.

In the measurement of time both systems agree. There are the hour, the minute, and the second. 1 hr. = 60 min. 1 min. = 60 sec. Length does not behave quite so nicely, as the accompanying tables show. The Metric system

UNITS OF LENGTH

British System	Metric System
1 mile (mi.) = 1,760 yards (yd.)	1 kilometer (km.) = 1,000 meters (m.)
1 yard = 3 feet (ft.)	1 meter = 100 centimeters (cm.)
1 foot = 12 inches (in.)	1 centimeter = 10 millimeters (mm.)



is formulated on the basis of units differing by multiples of 10 as in our money system. The relationship between different members of the two systems is interesting:

1 in. = 2.54 cm.

One meter is a little greater than a yard. The exact relationship you may find for yourself.

French Racetrack	Kentucky Derby Racetrack
Length = 2 km.	Length = $1\frac{1}{4}$ mi.
= 2,000 m.	= $\frac{5}{4} \times 1{,}760 = 2{,}200$ yd.
= 200,000 cm.	= $3 \times 2{,}200 = 6{,}600$ ft.

Speed, involving a combination of the two units, can be expressed in a great many ways, all of which are equivalent. Thus your speed from home to school was $\frac{1{,}320}{11}$ yd. per min., or $\frac{1{,}320 \times 3}{11} = 360$ ft. per min., or $\frac{360}{60} = 6$ ft. per sec. This may be extended to so many meters per second or kilometers per hour, etc.

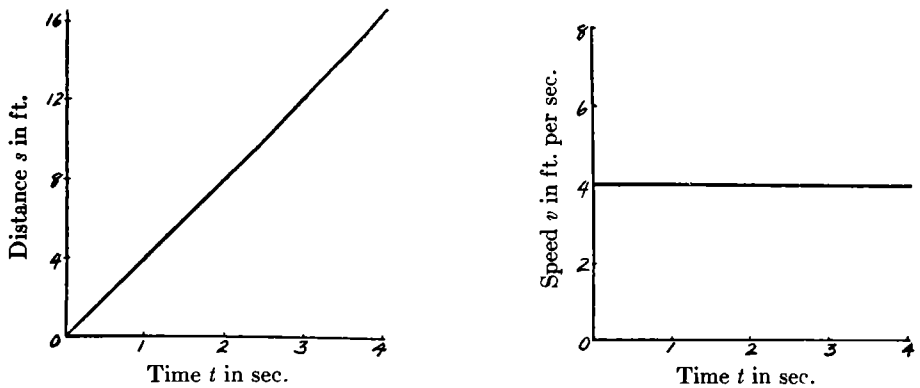
BUT NOW we must examine what we mean by this speed of 6 ft. per sec. On the way over to school you may have stayed for a minute with a friend. Then in order to get to class on time you hurried and maybe ran the last 100 yd. At any rate, whatever the cause, you did not cover 6 ft. in each second. Your speed was not 6 ft. per sec. each instant you were on the way. The 6 ft. per sec. is the average speed derived by dividing the total distance by the total time. You would have got to school in exactly the same time if you

had covered 6 ft. in each second; i.e., if you had a uniform speed of 6 ft. per sec. Your actual speed at different instants, i.e., your instantaneous speed, may have varied from zero, at the instants when you were delayed, to 12 ft. per sec. in the instants when you hurried. We must, therefore, be careful to notice when we speak of speed whether it is a UNIFORM SPEED, an AVERAGE SPEED, or an INSTANTANEOUS SPEED.

SUPPOSE, somewhat contrary to Shakespeare's lines,

> Love goes to love as schoolboy from his books;
> Love from love toward school with heavy looks,

you go home from school, a distance of 1,320 yd., with a slower but uniform speed of 4 ft. per sec.



Graphical representation of a uniform or constant speed of 4 ft. per sec.

1, 2. What is your instantaneous speed at the end of the first second? Fifth second?§*

3. What is your average speed from school to home?§

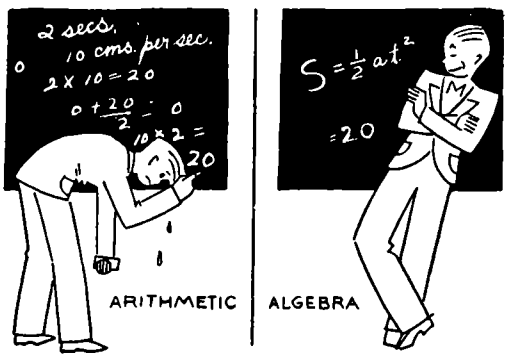
4. How long does it take you to get home, a distance of 1,320 yd.?§

BUT NOW WATCH YOUR STEP! It should never be forgotten that most answers contain not only a number but also a unit or units. In such cases the *units* in which the answer is expressed are *just as important as the number.* What would be the use of saying that your homeward speed was 4? You would naturally ask 4 what? 4 ft. per sec.? 4 yd. per sec.?

DON'T BE UPSET because we are going to introduce you to some simple algebra; you will find it a great labor-saver. As an aid to memory it is fre-

* All questions asked in the text are answered at the bottom of the following page and are indicated by §.

quently convenient to express relationships between different quantities in symbolic notation. Thus, in the example of your homeward journey from school which was done with uniform speed, the distance from home to school is equal to the uniform speed multiplied by the time taken. For distance (or length) the symbol s will be used; for time, t, and for speed, v. Thus for uniform speed:

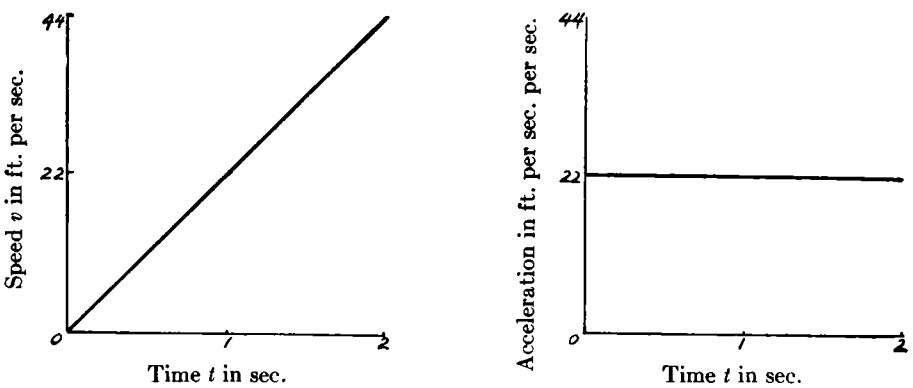


$$s = vt$$

The *average speed* from home to school was obtained by dividing the total distance s (1,320 yd.) by the time taken t (11 min.). It is convenient to designate this average speed by placing a bar across the letter v, as $\bar{v}$:

$$\bar{v} = \frac{s}{t}$$

LET US turn our attention to a more usual method of traveling, namely, motoring. Every automobile worthy of the name has its speedometer. Which speed (instantaneous, average, uniform) does the speedometer measure?§ The speedometer is calibrated in miles per hour. Your journey to school could probably be accomplished in 2 min. if you are not unfortunate



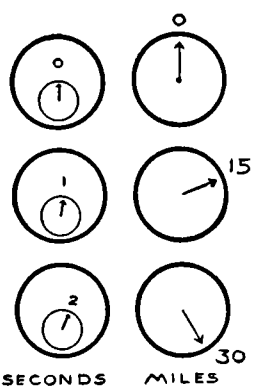
Graph of uniform acceleration of 22 ft./sec.²

enough to be held up by a stop-light. In this case your average speed is $\frac{1{,}320}{120} = 11$ yd. per sec. Let us suppose that on one occasion no stop-light interfered so that you went to school with a uniform speed of 11 yd. per sec.

§ Answers to questions on the previous page: (1, 2) Instantaneous speed: 4 ft. per sec.; (3) Average speed: 4 ft. per sec.; (4) Time taken: 990 sec., i.e., 16 min. 30 sec.

Each second you were in motion brought you 11 yd. nearer school. On another day you are less fortunate and a stop-light brings you to a halt. Your speed at this instant is zero. The light turns green, so you put your foot on the accelerator to start to increase your speed.

At the beginning of the second your speed is zero and at the end of this second it is, we will say, 15 mi. per hr. (15 MPH). Now 60 MPH = 88 ft. per sec., so that 15 MPH $= \frac{88}{60} \times 15 = 22$ ft. per sec. At the end of the second second it is, perhaps, 30 MPH. Then you relax pressure on the accelerator and cruise along with uniform speed. During the 2 sec. you have been increasing your speed; this is called an ACCELERATION. ACCELERATION IS THE RATE OF CHANGE OF SPEED WITH TIME. In this particular case we are dealing with a *uniform acceleration,* for in each of the 2 sec. you increased your speed by the same amount, 15 MPH. Thus your acceleration is 15 MPH every second. We shall limit our discussion to the case of uniform acceleration for which we shall use the symbol a:



$$\text{Acceleration} = \frac{\text{change in speed}}{\text{time taken}}; \qquad \text{i.e., } a = \frac{v_c}{t},$$

where v_c is the change in speed.

In the first second the change in speed was from 0 to 15 MPH, while in the second second it was from 15 to 30 MPH. In each case the acceleration is $\frac{15 \text{ MPH}}{1 \text{ sec.}}$, i.e., 15 MPH per sec. *Notice that a time unit must be used twice.* Now 15 MPH is 22 ft. per sec., so that your acceleration is $\frac{22 \text{ ft. per sec.}}{1 \text{ sec.}}$, i.e., 22 ft. per sec. per sec., or, more conveniently, 22 ft. per sec.^{2}.

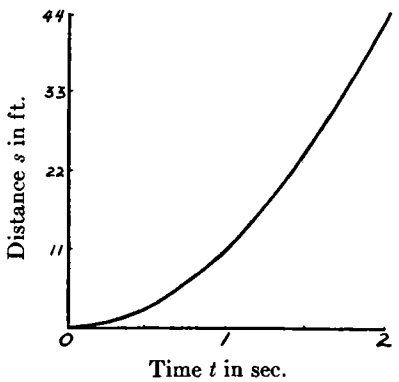
But how much distance was covered in the first second? Not 22 ft., for the speed was 0 ft. per sec. at the beginning of the second. However the *average* speed during this first second was $\frac{0+22}{2}$, or 11 ft. per sec.; $\bar{v} = 11$ ft. per sec. You will recall that the average of two quantities is their sum divided by 2.

$$\therefore \text{ Average speed} = \frac{\text{initial speed} + \text{final speed}}{2}, \qquad \bar{v} = \frac{v_i + v_f}{2},$$

so that the distance covered in the first second, given by $s = \bar{v}t$, is 11 ft. per sec. $\times$ 1 sec., namely, 11 ft.

§ Answer to question on the previous page: Instantaneous.

Let us consider the conditions during the second second. At the beginning of this second the speed was 22 ft. per sec. and at the end 44 ft. per sec., so that the average speed during this second is $\frac{22+44}{2}=33$ ft. per sec. The distance covered in this second will be 33 ft.



Hence in the 2 sec. a total distance of 11+33 ft. = 44 ft. is covered. But this answer might just as well have been calculated by taking the average speed during the first 2 sec. For the first 2 sec.

$$\bar{v}=\frac{v_i+v_f}{2}=\frac{0+44}{2}=22 \text{ ft. per sec.},$$

so that for the distance covered during the first 2 sec.

$$s=\bar{v}t=22\times 2=44 \text{ ft.}$$

It is well to notice that in calculating the distance covered during a certain time the average speed must be calculated for this same time.

WE HAVE seen that acceleration is given by $a=\frac{v_c}{t}$, where v_c, the change in speed, is equal to the final speed minus the initial speed:

$$v_c=v_f-v_i\ .$$

The distance moved is given by $s=\bar{v}t$, where $\bar{v}$, the average speed, is equal to the final speed plus the initial speed divided by 2:

$$\bar{v}=\frac{v_f+v_i}{2}\ .$$

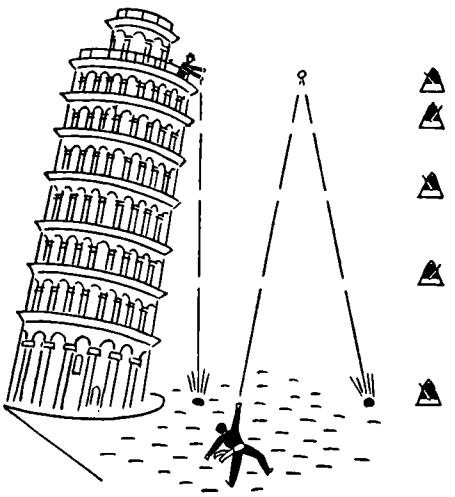
Now in most of the problems that we shall try the initial speed, v_i, will be zero. This makes things much easier, for IF $v_i=0$, then $a=\frac{v_f}{t}$ or $v_f=at$, and $\bar{v}=\frac{v_f}{2}$. In this case

$$\text{Distance moved } (s)=\bar{v}t=\frac{v_f}{2}\times t=\frac{at}{2}\times t$$

$$=\tfrac{1}{2}at^2$$

THE MOST common example we have of uniform acceleration is that of a freely falling body. As Galileo showed at Pisa, all bodies, irrespective of their weight, fall with the same acceleration, due to the attractive force of gravity. This acceleration due to gravity is designated by g; it is just a special acceleration:

$g = 32$ ft. per sec.^{2} or 980 cm. per sec.^{2}



WHEN AN automobile is coming to a stop, or a stone is thrown vertically upward, its speed is decreasing so that its motion is retarded. This decreasing of speed with time is called a retardation or a negative acceleration.

EXAMPLES

1. An automobile starts from rest and accelerates for 2 sec., reaching a speed of 45 MPH (66 ft. per sec.). Find
   a) The average speed during this period.
   b) The acceleration.
   c) The total distance traveled during the 2 sec.
   d) The instantaneous speed at the end of 1 sec. (the speedometer reading).

$$\text{Average speed: } \bar{v} = \frac{v_c}{2} = \frac{0+66}{2} = 33 \text{ ft. per sec.}$$

$$\text{Acceleration: } a = \frac{v_c}{t} = \frac{66}{2} = 33 \text{ ft. per sec.}^2$$

$$\text{Distance traveled: } s = \tfrac{1}{2}at^2 = \tfrac{1}{2} \times 33 \times 4 = 66 \text{ ft.}$$

Instantaneous speed at the end of the first second: Since the automobile is starting from rest, the instantaneous speed at the end of the first second is the same as the change in speed v_c during the second. We can calculate the change in speed, v_c, during this second:

$$a = \frac{v_c}{t} \text{ or } v_c = at = 33 \times 1 = 33 \text{ ft. per sec.}$$

Problem.—Calculate the distance traveled during the first second and also during the second second. (*Answers:* 16.5 ft.; 49.5 ft.)

2. An automobile going with a speed of 60 MPH (88 ft. per sec.) puts on the brakes and comes to rest in 4 sec. Find the value of the retardation and also the distance traveled while the brakes are on before coming to rest.*

* We shall later find it an advantage to use as our fundamental units feet or centimeters, and seconds, i.e., to express speed in ft. per sec. or cm. per sec., and accelerations in ft. per sec.^{2} or cm. per sec.^{2}.

Initial speed = 88 ft. per sec. Final speed = 0

Change in speed $v_c = v_f - v_i = 0 - 88 = -88$ ft. per sec.

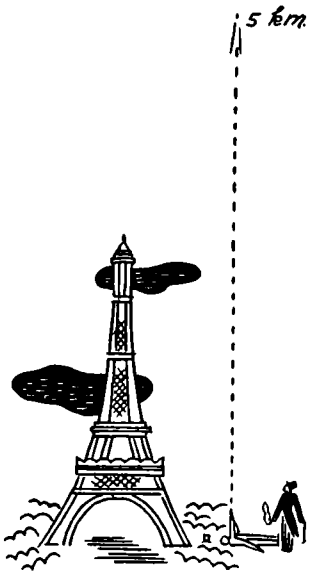
Retardation $a = \frac{v_c}{t} = -\frac{88}{4} = -22$ ft. per sec.^{2}

Distance traveled $s = \frac{1}{2}at^2 = -\frac{1}{2} \times 22 \times (4)^2 = -\frac{1}{2} \times 22 \times 16$ ft.

$= -176$ ft.

The minus sign need not worry us; it comes about due to negative acceleration or retardation. The distance traveled is 176 ft.

3. A bullet is shot vertically upward with a speed of 10,000 cm. per sec. Find how high it rises and how long it is in the air.



NOTE.—Since the bullet is shot vertically upward, it will strike the ground at the same place as it left. In fact, it should re-enter the barrel of the gun with the same speed as it left it if the effect of wind or friction of the air is neglected.

Consider first only the motion upward from the time the bullet leaves the gun until it is momentarily at rest at the top of its flight.

$v_c = v_f - v_i = 0 - 10{,}000 = -10{,}000$ cm. per sec.

In the upward motion the speed is decreasing uniformly with time, i.e., there is a retardation:

$a = -g = -980$ cm. per sec.^{2}

$a = \frac{v_c}{t}$ or $t = \frac{v_c}{a}$

$t = \frac{-10{,}000}{-980} = +\frac{10{,}000}{980} = 10.2$ sec.

= time required for bullet to come to rest, i.e., to attain its maximum height.

Distance traveled:

$s = \frac{1}{2}at^2 = \frac{1}{2} \times (-980) \times (10.2)^2 = -\frac{1}{2} \times 980 \times 104$ cm.

$= -51{,}000$ cm.

Thus the vertical height which the bullet reaches is 51,000 cm.

The time required for the downward flight will be exactly the same as that required for the upward flight. Hence the total time between leaving the gun and striking the ground will be $2t = 20.4$ sec.

Check by calculation that the time required for the bullet to reach the earth from the top of its flight is 10.2 sec.

SUMMARY

Average speed: $\bar{v} = \frac{s}{t}$; $s = \bar{v}t$

Constant or uniform speed: $s = vt$

Uniformly accelerated motion: $a = \frac{v_c}{t} = \frac{v_f - v_i}{t}$

$\bar{v} = \frac{v_i + v_f}{2}$; $v_c = at$

Uniformly accelerated motion when object starts from rest: $v_i = 0$

$$v_c = v_f \; ; \qquad \bar{v} = \frac{v_f}{2}$$

$$a = \frac{v_f}{t}$$

$$s = \frac{v_c t}{2} = \tfrac{1}{2} a t^2$$

Also since $v_f = at$, i.e., $t = \frac{v_f}{a}$

$$s = \tfrac{1}{2} a t^2 = \tfrac{1}{2} a \times \frac{v_f^2}{a^2} = \frac{1}{2} \frac{v_f^2}{a}$$

$$\therefore \quad v_f^2 = 2as$$

PROBLEMS

1. Light travels with the enormous speed of 186,000 mi. per sec.
   a) Find the time required for light to travel from the moon to the earth, a distance of 240,000 mi.
   b) Find the time required for light to travel from the sun to the earth, a distance of 92,000,000 mi.
   c) The star nearest to the earth is Alpha Centauri and it takes light nearly $4\frac{1}{2}$ years to reach the earth from the star. Find the distance in miles of the star from the earth.

2. In the Settle-Fordney stratosphere flight the balloon descended from 62,000 to 800 ft. in 60 min. Find the average speed of descent in ft. per min. and in ft. per sec.

3. Suppose that a rocket is shot from the earth directly at the moon, with a speed of 10,000 ft. per sec. and that this speed remains constant (quite impossible). Find the time of flight in hours and minutes.

4. A crack train goes from Chicago to New York, a distance of about 900 mi., in 18 hr.
   a) What is the average speed of the train in MPH?
   b) Would this be a constant speed?
   c) Would this be a uniform speed?

5. Sound travels with a uniform speed of 1,100 ft. per sec. in calm air all at the same temperature. How long does it take for an echo to be heard from a mountain side $\frac{1}{2}$ mi. away?

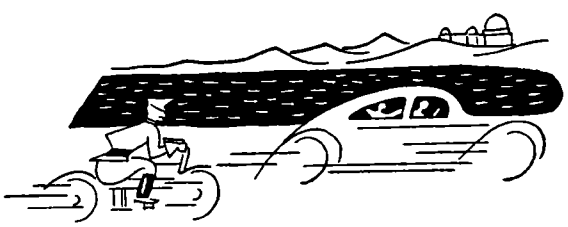
6. An automobile starts from rest and acquires a speed of 500 cm. per sec. in 1 sec.
   a) What is its acceleration during this second?
   b) What is its average velocity during this second?
   c) How far does the automobile travel in this second?

7. A sprinter takes 10.3 sec. to run the 100-m. straight dash. If he takes 4 sec. over the first 30 m., find
   a) His average speed during the first 4 sec.
   b) His average acceleration during the first 4 sec.
   c) His average speed during the last 6.3 sec.

8. An automobile having a speed of 500 cm. per sec. increases its speed to 800 cm. per sec. in 2 sec.
   a) What is its acceleration during the 2 sec.?
   b) What is its average velocity during the 2 sec.?
   c) How far does the automobile travel in the 2 sec.?

9. An automobile starts from rest and acquires a speed of 30 MPH in 3 sec.
   a) Find its acceleration in MPH per sec. and in ft. per sec.^{2}.
   This automobile then approaches a stop-light and comes to rest in 1 sec.
   b) Find its retardation in ft. per sec.^{2}.

10. An automobile starts from rest and accelerates over a distance of 300 ft. until its speed is 100 ft. per sec. (approximately 69 MPH). It continues with this speed for 5 min., when the driver spots a cop coming after him and immediately puts on his brakes. He comes to rest in 4 sec. Find



   a) The average speed of the car during the time it is accelerating.
   b) The time taken to acquire this speed of 100 ft. per sec.
   c) The value of the acceleration.
   d) The distance traveled during the 5 min. in which the car is going with the uniform speed.
   e) The retardation during the 4 sec. after the cop has been seen.
   f) The distance traveled during the 4 sec.

11. Represent by graphs of speed against time, and distance against time, the motion of the automobile in problem 10.

12. A stone is dropped from a building 400 ft. high. How long does it take to reach the ground?

13. A stone is dropped into a well 400 ft. deep. How long is it before the stone hits the water? Before the splash is heard?

14. A person skiing down a hill has an acceleration of 100 cm. per sec.^{2}. If he started from rest, find
    a) His velocity at the end of the first second.
    b) His velocity at the end of the second second.
    c) His average speed during the first second.
    d) How far he travels down the hill in the first second.
    e) His average speed during the first 2 sec.
    f) How far he travels down the hill in the first 2 sec.
    g) His average speed during the second second.
    h) How far he travels in this second second.

15. A bullet is shot out of a rifle with a speed of 30,000 cm. per sec. If the bullet takes $\frac{1}{100}$ sec. to travel the length of the barrel, find its acceleration while in the barrel.

16. If the bullet of problem 15 is shot vertically upward, find
    a) How long the bullet takes before coming to rest at the top of its flight.
    b) How high it rises vertically.
    c) The total time the bullet is in the air.

17. When driving a car, a record is kept of the distance traveled in each 15 min. The accompanying table gives such a record:

Time			Distance
Hours		Minutes	Miles
9	:	00	0
	:	15	6
	:	30	15
	:	45	24
	:	60	36

Plot a graph of distance traveled against time.
    a) What is the average speed between 9:00 and 9:15?
    b) What is the average speed between 9:15 and 9:30?
    c) What is the average speed between 9:30 and 9:45?

d) What is the average speed between 9:45 and 9:60?

e) What is the instantaneous speed at 9:10?

Plot a graph of speed in MPH against time, assuming uniform speed during the 15-min. intervals.

18. A stone is dropped from rest from a very high building.
    a) Find the distance in feet the stone falls in the first second, the first 2 sec., the first 3 sec., and the first 4 sec.
    b) Find the distance the stone falls in the second second, the third second.
    c) What is the simple relationship between the distance fallen in the first second, the second second, the third second?
    d) From this relationship find how far the stone falls in the ninth second.

19. A stone is dropped from a building 576 ft. high. Plot a graph of
    a) Speed of stone against time.
    b) Distance fallen against time.

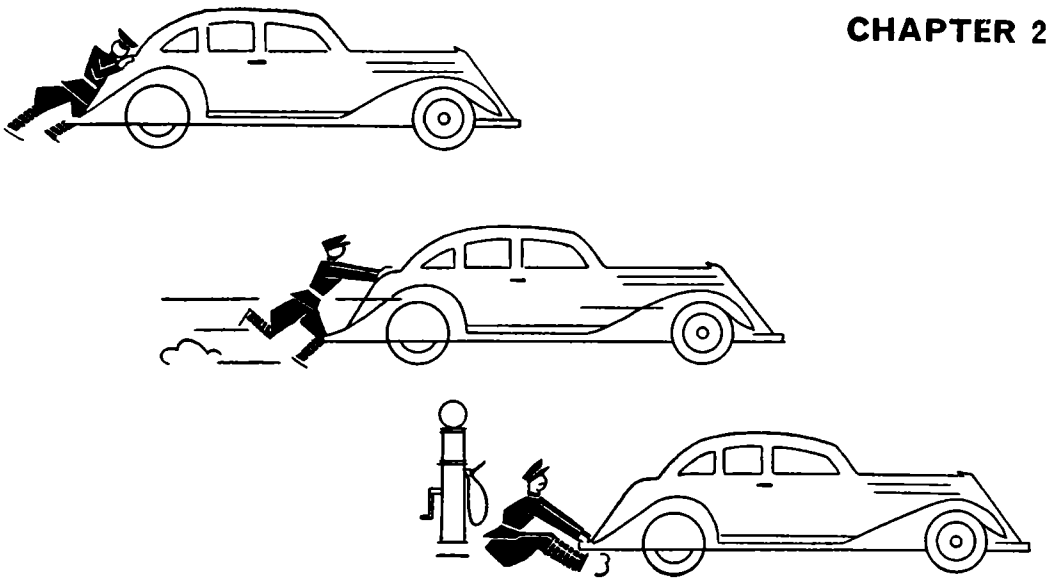
    Make up a table:

Time	1 sec.	2 sec.	etc.
Speed			

also

Time	1 sec.	2 sec.	etc.
Distance fallen			

CHAPTER 2



NEWTON'S FIRST TWO LAWS OF MOTION

Nature and Nature's laws lay hid in night,
God said "Let Newton be"; and all was light.

IT IS somewhat surprising that the laws relating to the motion of bodies were not discovered until the seventeenth century. This was not because people had not thought about natural phenomena; before 100 B.C. Eratosthenes of Alexandria had measured the circumference of the earth and Hipparchus had discovered the precession of the equinoxes. Such problems required real intellectual effort, yet the problems relating to the motion of bodies were not successfully attempted until 1600 by Galileo and Newton.

SUPPOSE one day you are out driving and your car runs out of gas. The only thing to do is to push it to the nearest gas station. You realize quite soon that the push required to get the car started is much greater than that required to keep it going, provided, of course, you encounter no hills. If at some point you decide to make it go faster you will have to give an extra push to accomplish this. Then in order to stop at the gas station you have to pull back on the rear bumpers. If this happens often you are probably glad if you have a light car, or, as Galileo would have put it, one with a small *inertia.* Then the push required to set the car in motion is smaller than for heavier cars having a greater inertia.

THE reason you have to push, once the car is in motion, is because the wheel bearings are not entirely frictionless and the air offers resistance. Had

the bearings been entirely frictionless and the air somehow offered no resistance, then once the car was in motion it would have kept that motion *undiminished but in a straight line.* Turning a corner requires an extra push just as speeding it up does.

THUS TO alter the motion of an object, start, stop, speed it up, slow it down, as well as to change its direction, a push or force is required. The earth and the other planets would need no force to keep them moving in straight lines, but do need a force to make them swing around the sun.

The amount of the push, i.e., FORCE, required to alter the motion of your car depends on the INERTIA of the car. The quantitative measure of inertia is *mass.* The standard mass in the Metric system is the *gram* (gm.), this being the mass of a platinum-iridium block which is kept at Paris. Masses can be compared by an ordinary chemical balance and hence copies of this standard mass are readily available, not of course made of platinum-iridium.

NEWTON summarized the relationship between forces and motion in his first two LAWS OF MOTION:

Law I. Every object persists in its state of rest or of uniform motion in a straight line unless acted on by some external force.

Law II. The measure of a force which has acted on an object to alter the state of its motion is equal to the product of the mass of the object and the acceleration produced.

Law I is a statement of Galileo's principle of inertia which we have already discussed.

Law II:

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$F = ma$$

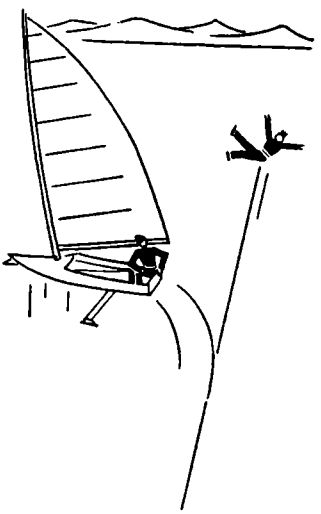
Using this law, we can get our unit of force. In order not to get too confused in our units, we shall use only the *Metric system.** This system is also called the *c.g.s. system* because its fundamental units are the *centimeter, gram,* and *second.* The *unit of acceleration is 1 cm. per sec. per sec.* The unit of force in the c.g.s. system is called the **dyne** and is the **force required to give a mass of 1 gm. an acceleration of 1 cm. per sec. per sec.**

NOW WE stated that you have to exert a push on the automobile to change either the magnitude, i.e., amount, of its motion, or the direction in which it was going. There is a single word, **velocity,** which includes the two factors, magnitude and direction of motion. Such a quantity, which includes

* This system is the one adopted in this country by an act of Congress in 1890.

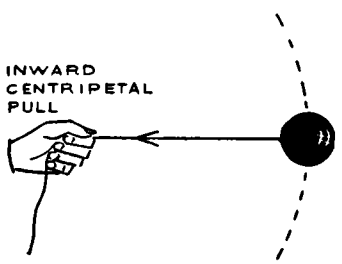
both *magnitude* and *direction*, is called a **vector**. Later we shall learn a new kind of algebra for adding vectors. From now on we shall let the symbol v, previously used for speed, stand for velocity. Velocity includes speed. Speed is just the magnitude of the velocity without any reference to direction.

The acceleration we discussed in the first chapter is the acceleration of any object moving along a straight line, a *linear acceleration*. This acceleration was produced by changing the magnitude of the velocity, the speed of the object, without changing the direction of motion. But we also have an acceleration if the velocity is *constant in magnitude* but is *changing in direction*. You must have noticed the tendency to slide outward on an automobile seat when rounding a sharp corner with the speedometer reading consistently at, say, 40 MPH. You were subject to an acceleration even though the magnitude of your velocity remained the same, 40 MPH.



The result of not holding on: an iceboat tragedy

You will have to take our word for it that an object having a constant speed v moving in a circle of radius r has an acceleration of v^2/r. So that if the object has a mass m, the force on it is the product of its mass and its acceleration, mv^2/r. An object moving with a constant speed in a circle has the direction of its velocity changing uniformly, hence this acceleration, v^2/r, is a uniform or constant acceleration. A railway train or street car which has no steering apparatus goes round a bend because the outside rails push sideways on the train. Is there any such push involved when an automobile turns a corner?§ Consider what happens when the road is icy.



EXAMPLE. We shall calculate how much tension there is in a string when a stone tied to one end is swung around in a circle. The string is pulling inward on the stone all the time causing it to revolve in a circle. If the string suddenly breaks, what happens to the stone?§§ Suppose the length of the string is 28 cm. and that the stone of mass 100 gm. goes around twice per second.

The distance the stone travels when it goes once around the circle is the circumference of the circle, $2\pi r$, i.e., $2\times\pi\times 28$ cm., i.e., 176 cm. ($\pi = \frac{22}{7}$). But the stone goes around twice every second, so it goes a distance of 2×176, or 352 cm. every second. Thus the speed, or magnitude of the velocity of the stone, is 352 cm. per sec.

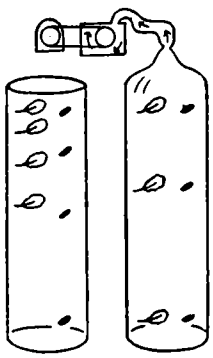
$$\text{Tension in string, i.e., force: } F = \frac{mv^2}{r} = \frac{100\times 352\times 352}{28} = 442{,}500 \text{ dynes}$$

This force, or tension in the string, is called the centripetal force. It is an inward pulling force.

These two types of acceleration—linear acceleration and acceleration in a circle—are included when we define **acceleration as rate of change of velocity with time.** Almost all of our work will be on linear accelerations such as were discussed in the previous chapter.

Weight is a force and its units are more familiar to you than the dyne.

Galileo showed by dropping two heavy objects from the top of the Leaning Tower of Pisa that they fell to the ground with exactly the same acceleration. Later Newton showed that a light object falls with this same acceleration if the resistance of the air be removed by placing the object in a vacuum. This acceleration, called g, is equal to 980 cm. per sec.^{2} at our latitude. It is slightly greater at the poles and slightly less at the Equator.



Heavy weight and feather falling in air in a vacuum

Imagine holding some 1 gm. mass in your hand—a penny* will do well enough. If you remove your hand, the penny falls toward the earth with an acceleration of 980 cm. per sec.^{2}. Had it fallen with an acceleration of 1 cm. per sec.^{2}, the force on the penny would have been 1 dyne; but it falls with the acceleration of 980 cm. per sec.^{2}, so the force on it must be 980 dynes.

Now when the penny is resting in your hand, this force of 980 dynes is still present, a force which you have always called **weight,** in this case a 1 gm. weight (gm. wt.), the weight of the penny. Therefore the 1 gm. wt. is the same as 980 dynes of force. **Weight is the vertical force acting on every mass due to the attractive force of gravity.** A body of mass, m, gm., has a weight of mg dynes.

Weight = mass × acceleration due to gravity

$$w = mg$$

Thus 1 gm. wt. = 980 dynes, 10 gm. wt. = 9,800 dynes, etc. If the attractive force of gravity were suddenly to become zero, objects would no longer be accelerated toward the earth. When you let go of your penny it would not fall but would remain suspended in the air. The

What? No force of gravity!

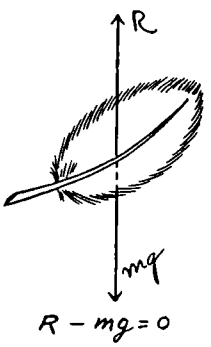
§ Answer to question on the previous page: Yes.

§§ Answer to question on the previous page: The stone moves off in a straight line in the direction in which it was going at the instant the string breaks.

* Actually a penny weighs about 3 gm.

penny, then, would have no weight (0 gm. wt.), but it would still have inertia or mass (1 gm. mass).

FROM YOUR everyday observations you know that a feather, or any light object, does not fall to the earth with the acceleration of gravity. It falls with a uniform velocity called its *terminal velocity*. This being the case, the total or resultant force acting on the feather must be zero. The downward force due to the weight of the feather must be balanced by the upward force due to the resistance of the air. The same is true of a parachute. Why is it that when the resistance of the air balances the weight of an object it moves downward and not upward?§ Remember that in using Newton's Law, if more than one force is acting on an object, the force F is the resultant of the forces. You must always put the force F in dynes, the mass in gm., and the acceleration in cm. per sec.^{2}. Then you will avoid trouble!



$$F = m \times a$$

dynes = gm. mass × cm. per sec.^{2}

YOU MIGHT quite naturally ask to what this attractive force of gravity is due? In other words, why do objects fall with this acceleration of 980 cm. per sec.^{2}? The ultimate why can never be given in physics, but an explanation was given by Newton about the same time he enunciated his Laws of Motion.

The story begins with the last and greatest of the astronomers who made observations on the stars and planets without a telescope. The man, Tycho Brahe, was a peculiar genius not above practicing astrology when it was to his advantage. Nevertheless the data he obtained on the apparent motions of the planets among the stars enabled Kepler to determine the laws relating to the motion of the planets around the sun.

Kepler was as persevering and intellectually upright in unraveling Tycho's data as Tycho had been in recording them. After twenty years of effort he summarized the results into three simple laws. These laws, which will be found in any astronomy book, are themselves very important, but to us they are especially important inasmuch as they led Newton to formulate his famous **Law of Gravitation: Every particle in the universe attracts every other particle with a force which is proportional to the product of their masses and which varies inversely as the square of the distance between them.** Thus if we have two particles of masses m_1 and m_2 gm. a distance r cm. apart, the force with

which each attracts the other is proportional to $\frac{m_1 \times m_2}{r^2}$. This may be written $F = \text{a constant} \times \frac{m_1 \times m_2}{r^2}$. The constant, usually denoted by G, is known as the constant of gravitation: $F = G \times \frac{m_1 \times m_2}{r^2}$. The value of G has to be determined experimentally. The first attempt was made by Cavendish. Many attempts have been made since the time of Cavendish to determine G. The accepted value is 6.67×10^{-8} c.g.s. units. If we place two 1 gm. masses so that there is 1 cm. between their centers, they attract each other with a force of 6.67×10^{-8} dynes. Newton was able to show, after considerable mathematical difficulty, that if the masses are spheres, the centers of attraction are the centers of the spheres.

EXAMPLE. Find the force of attraction between two spherical objects, one having a mass of 100 gm. and the other of 800 gm., if the distance between their centers is 5 cm.

$$m_1 = 100;\ m_2 = 800;\ r = 5$$

$$F = G\,\frac{m_1 \times m_2}{r^2} = 6.67 \times 10^{-8} \times \frac{100 \times 800}{25} = 2.13 \times 10^{-4} \text{ dynes}$$

This is so small a force that it would require quite delicate apparatus to detect it.

THE FORCE of gravity on an object is, then, nothing more than the gravitational attraction between the object and the earth. The earth is so huge compared to any objects we lift around on the earth that its attractive force "swamps" all the others. But now we should be able to calculate the mass of the earth since we know that a mass of 1 gm. is pulled toward the earth with a force of 980 dynes. All we need is the radius of the earth. This we shall leave for you to do in problem 26.

EXAMPLES

1*a*. A dog pulls a sled having a mass of 100 kg. over smooth ice. The dog pulls with a horizontal force of 10,000 gm. wt. Find the acceleration of the sled and the distance moved in 2 sec. if the sled starts from rest.

Pulling force: $F = ma$. $\quad \therefore \quad a = \frac{F}{m} = \frac{10{,}000 \times 980}{100 \times 1{,}000} = 98.0$ cm. per sec.^{2}

(F must be in dynes. It is $10{,}000 \times 980$ dynes.)

Distance moved in 2 sec.:

$$s = \tfrac{1}{2}at^2 = \tfrac{1}{2} \times 98 \times 4 = 196 \text{ cm.}$$

(What is the velocity at the end of the 2 sec.? What is the average velocity during the 2 sec.?) (*Answers*: 196 cm. per sec.; 98.0 cm. per sec.)

§ Answer to question on the previous page: Newton's First Law—the object is already moving down and continues with this motion.

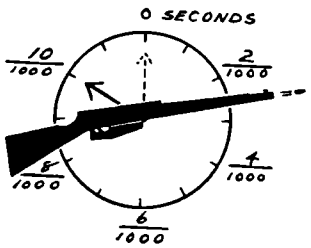
1*b*. The sled now comes to a bare spot of ground where the force of friction between the sled and the ground is 10,000,000 dynes. Find the horizontal force in dynes and gm. wt. which must now be applied to the sled to give it an acceleration of 98 cm. per sec.^{2}.

$$F = ma = 100{,}000 \times 98 = 9{,}800{,}000 \text{ dynes}$$

This is the force necessary to produce the acceleration on smooth ice, but an additional force of 10,000,000 dynes must be applied to overcome the force of friction.

$$\therefore \text{ Total force} = 9{,}800{,}000 + 10{,}000{,}000 = 19{,}800{,}000 \text{ dynes}$$

$$= \frac{19{,}800{,}000}{980} = 20{,}200 \text{ gm. wt.}$$



2*a*. A bullet whose mass is 4 gm. is fired from a gun whose barrel is 80 cm. long and takes $\frac{1}{100}$ sec. to travel down the barrel. Find the average velocity of the bullet while coming down the barrel, the velocity with which the bullet comes out of the gun, and the force exerted on the bullet while traveling down the barrel.

Average velocity: $\bar{v} = \frac{s}{t} = \frac{80}{\frac{1}{100}} = 80 \times 100 = 8{,}000$ cm. per sec.

Velocity change: $\left(\bar{v} = \frac{v_c}{2}\right)$ or $v_c = 2\bar{v} = 16{,}000$ cm. per sec.

Velocity on leaving barrel: v_f

Velocity change = final velocity − initial velocity

$16{,}000 = v_f - 0$

$v_f = 16{,}000$ cm. per sec.

Acceleration: $a = \frac{v_c}{t} = \frac{16{,}000}{\frac{1}{100}} = 1{,}600{,}000 = 1.6 \times 10^6$ cm. per sec.^{2}

Force exerted: $F = ma = 4 \times 1.6 \times 10^6 = 6.4 \times 10^6$ dynes
(This is the force on the bullet while traveling down the barrel.)

2*b*. The bullet penetrates 4 cm. into a block of wood before coming to rest. Find the resisting force exerted by the block of wood on the bullet.

We find the retardation, $-a$, by calculating the time the bullet takes to come to rest in the block of wood:

$$\left(s = \frac{v_c t}{2}\right), \qquad t = \frac{2s}{v_c}, \qquad t = \frac{2 \times 4}{16{,}000} = \frac{1}{2{,}000} \text{ sec.}$$

$$\text{Retardation: } a = \frac{v_c}{t} = \frac{v_f - v_i}{t}.$$

But $v_f = 0$ since the bullet comes to rest after going 4 cm.

$$\therefore \quad a = \frac{-16{,}000}{\frac{1}{2{,}000}} = -16{,}000 \times 2{,}000 = -32{,}000{,}000 \text{ cm. per sec.}^2$$

Retarding force: $F = ma = 4 \times -32{,}000{,}000 = -1.28 \times 10^8$ dynes

The negative sign indicates that the force is resisting the motion, i.e., producing a retardation.

PROBLEMS

1. A person has a mass of 70 kg. and a speed of 800 cm. per sec.
   a) Find the mass of the person in lb. and his speed
   b) In ft. per sec. and
   c) In mi. per hr.

   1 lb. = 453 gm.; 1 ft. = approx. 30 cm.; 1 kg. = 1000 gm.

2. *a*) What force in dynes do you exert when you hold up a bag of sugar having a weight of 1,000 gm.?
   b) What force in dynes do you exert when you hold up a bag of sugar having a mass of 1,000 gm.?

3. A car is moving along a straight road with a constant velocity of 500 cm. per sec.
   a) Are there any forces acting on the automobile?
   b) What is the total or resultant force acting on the automobile?

4. A boy cycles round a bend with a constant speed of 10 MPH.
   a) Is his velocity constant?
   b) Does the boy have an acceleration?

5. A drop of rain whose mass is 0.5 gm. falls with a constant velocity of 8 m. per sec.
   a) What is the acceleration of the raindrop?
   b) What is the force on the drop due to air resistance?

6. An object having a mass of 5 gm. falls vertically in the air and finally reaches a velocity such that the force due to air resistance balances the weight of the object. Find
   a) The resultant force in dynes acting on the object.
   b) The acceleration of the object.
   c) The force in dynes due to air resistance when the object moves with constant velocity.

7. A golf ball whose mass is 30 gm. lies at rest on the ground. It is struck by a golf club and given a speed of 1,000 cm. per sec. The golf club is in contact with the ball for $\frac{1}{2}$ sec.
   a) Find the acceleration of the ball when struck.
   b) Find the force exerted by the club on the ball.

8. An automobile traveling on a straight road speeds up from 400 cm. per sec. to 1,000 cm. per sec. in 4 sec.
   a) Find the acceleration of the automobile.
   If the mass of the automobile is 1,500 kg,
   b) Find the force exerted by the engine to produce this acceleration.

9. An automobile having a mass of 1,500 kg. is traveling along a straight road with a velocity of 3,000 cm. per sec. It comes to rest in 10 sec., when the brakes are applied.
   a) Find the value of the retardation.
   b) Find the force exerted by the brakes of the automobile.

10. A man descends vertically in a parachute with a uniform velocity of 15 MPH. If the man and parachute have a mass of 100 kg., find
   a) The acceleration of the parachute.
   b) The total or resultant force acting on the man.
   c) The force of the resistance of the air in kg. wt. and dynes.

11. An automobile whose mass is 500 kg. picks up from rest to a speed of 2,000 cm. per sec. in 5 sec. on a level road. Find
   a) The distance traveled in this time.
   b) The acceleration or "pick-up" of the car.
   c) The force, in dynes and in gm. wt., exerted by the engine. (Neglect the forces of resistance of the air and wheel bearings). If the resistance forces amount to 50 kg. wt., find
   d) The total force, in kg. wt., which the engine must exert to produce the acceleration.
   Find the force which the engine must exert when the car travels with a uniform velocity of 2,000 cm. per sec. along a straight level road.
   e) If the car is perfectly streamlined, i.e., offers no resistance to the air, and has frictionless bearings.
   f) If the car offers an air resistance of 50 kg. wt.
   If the driver now puts on the brakes and comes to rest in 3 sec., find
   g) The force exerted by the brakes. (Neglect any other resistance forces.)

12. A person whose mass is 70 kg. is skating on ice at a speed of 800 cm. per sec. He glides to a stop in 20 sec. Find
    a) How far he travels in the 20 sec.
    b) The resistance forces bringing him to rest. (What are these resistance forces?)

13. A bullet whose mass is 5 gm. is shot out of a rifle with a velocity of 15,000 cm. per sec. The bullet takes $\frac{1}{100}$ sec. to go down the barrel of the rifle.
    a) Find the acceleration of the bullet when going down the barrel.
    b) Find the force exerted on the bullet by the explosion of the powder.

14. The bullet described in problem 13 is stopped in a block of wood after penetrating 5 cm. into the wood.
    a) Find the value of the retardation.
    b) Find the resisting force of the wood.

15. What happens to a person skating on hard ice when he suddenly moves onto soft ice?

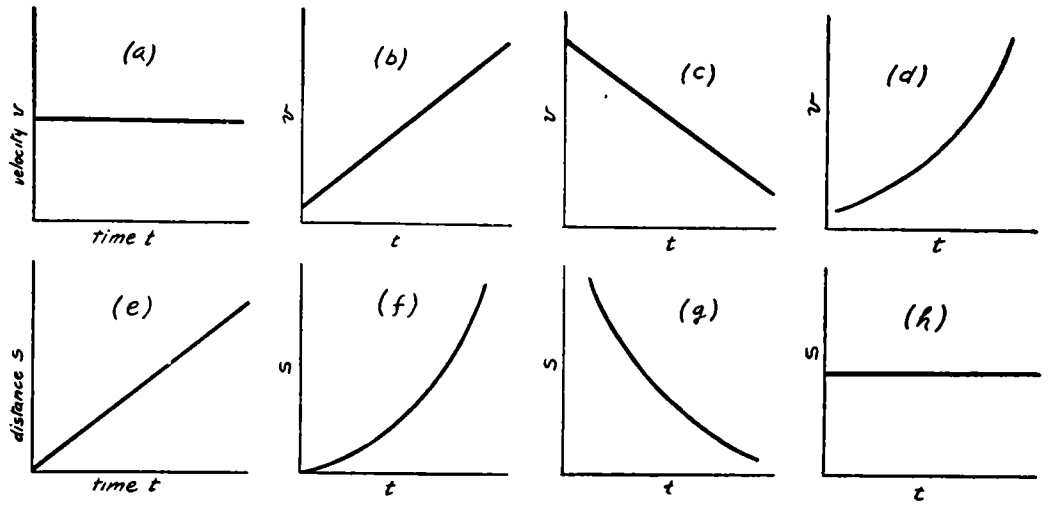
16. *a*) In which direction do you tend to fall when standing in a street car which suddenly starts?
    b) which stops quickly?

17. A person whose mass is 70 kg. is standing on an accurate weighing machine placed on the floor of an elevator. The weighing machine is calibrated in kg. What does the weighing machine indicate
    a) When the elevator is at rest?*
    b) If the rope of the elevator breaks and the elevator falls vertically with the acceleration due to gravity?
    c) If the elevator descends with an acceleration?
    d) If the elevator ascends with an acceleration?
    e) When the elevator is coming to rest at the bottom (i.e., has a retardation downward)?
    f) When the elevator is coming to rest at the top?

18. You are sitting on the edge of a horizontal turntable which has a radius of 2 m. and a velocity, constant in magnitude, of 1,000 cm. per sec. at the edge.
    a) Find the force of friction necessary to keep you on the table if your mass is 70 kg.
    b) If the force of friction were less than this amount, what would happen to you?

* Answer all these questions by zero, less than, equal to, or greater than 70 kg.

19. The accompanying graphs represent the motion of an automobile along different portions of a straight road. State in each case whether

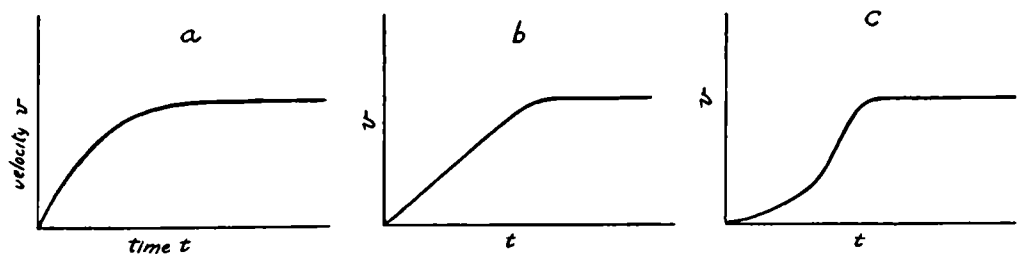


a) The automobile has an acceleration, retardation, uniform velocity, or is at rest.

b) There is a resultant force acting on the automobile.

20. What force do the rails have to exert on a street car of 10^5 kg. mass to make it turn a curve whose radius is 40 m.? The street car has a velocity, constant in magnitude, of 400 cm. per sec.

21. A man descends in a parachute from a balloon at rest at an altitude of several kilometers. Which of the graphs below would best represent his downward flight?



22. A boy catches a ball whose mass is 200 gm. which is coming toward him with a velocity of 150 cm. per sec. If in catching the ball he draws his hands back 10 cm., find

a) The average velocity of the ball while coming to rest.

b) The length of time the ball is in the boy's hands before it comes to rest.

c) The retardation of the ball.

d) The force exerted by the boy in bringing the ball to rest.
If he draws his hands back 20 cm. in catching the ball, find
e) The force exerted by him in bringing the ball to rest.

23. Two heavy balls are dropped at the same instant from two high windows 30 ft. apart. Do the balls keep 30 ft. apart on their way down?

24. Find the force of attraction between two spheres each having a mass of 200 gm., if their centers are placed 20 cm. apart.

25. If you weigh 100 kg. here on the earth's surface, what would your weight be if you were able to go up a distance equal to the radius of the earth?

26. Given the radius of the earth, 6.4×10^{8} cm. (about 4,000 mi.), and the data at the end of the chapter, calculate the mass of the earth.

27. Two particles having masses of 1 gm. and 1 gm. placed 1 cm. apart attract each other with a force of 6.67×10^{-8} dynes. What is the force of attraction
a) Between 1 gm. and 10 gm. placed 1 cm. apart?
b) Between 10 gm. and 10 gm. placed 10 cm. apart?
c) Between 1 gm. and 6.02×10^{27} gm. placed 1 cm. apart?
d) Between 1 gm. and 6.02×10^{27} gm. placed 6.4×10^{8} cm. apart?

CHAPTER 3

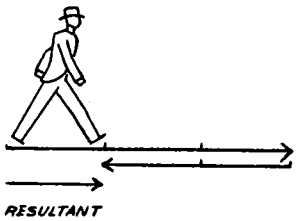
ADDITION OF VECTORS. PROJECTILES

IN THE discussion of Newton's Second Law we mentioned the case of a parachute descending with uniform speed even though there were forces acting on it. This was due to the fact that the two forces—the resistance offered by the air and the weight of the parachute and man—exactly balanced each other. In this case the forces were in the same vertical straight line but in opposite directions, their resultant being zero.

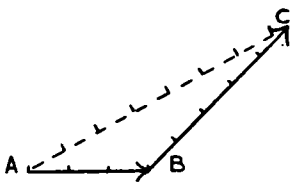
FORCES, then, like velocities, are vector quantities. They have both magnitude and direction. Displacements, accelerations, momenta, etc., are all *vector* quantities. On the other hand, quantities such as mass, volume, speed, etc., are called *scalars*. They have only magnitude. We learned to add scalar quantities in grammar school; we simply added numbers. But in adding vectors we have to take the directions into account, so it is not quite so easy as adding scalars. We shall now give a simple experimental demonstration of how vectors are added geometrically, using displacement as the vector quantity:

Suppose you take 3 steps E. and then 2 W. You arrive at a place 1 step E. from your starting-point. This stepping-out may be shown on a diagram.

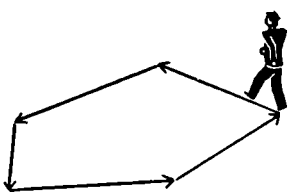
Any vector can be represented by a line, the *length* representing the *magnitude* of the vector and an *arrow* on the line representing the *direction* of the vector.* Any scale may be used in the diagram, but we shall here use 1 cm. to represent 1 step. Let us call the direction E. (+) and W. (−).† Then we are adding two vectors, +3 and −2. Their sum is +3−2, which is +1. The resultant of these two vectors is therefore a vector whose magnitude is 1 and whose direction is east, the latter because of the (+) sign.



This is quite easy, but what happens if the vectors are not in the same straight line? Suppose you take 3 steps E. and then 5 steps NE. How far are you from your starting-point? You can find out by actually measuring on the floor or geometrically by drawing a vector diagram. The vectors are placed in the correct direction with the *head of one touching the tail of the other.* The *resultant, AC,* is found by *measuring the length of the line joining the tail of the first vector to the head of the second.*‡ In either case the resultant is found by measurement to have a length of 7.4 steps in a direction approximately NE. by E. This means you would have landed at the same place had you taken 7.4 steps in a direction NE. by E. as you did by taking 3 steps E. and 5 NE.**



This method may be used for more than 2 vectors. The vector diagram is, then, a polygon instead of a triangle. If the vector polygon or triangle is closed (i.e., the head of the last vector just meets the tail of the first), the resultant is zero. In other words, if the vectors represent your steps, you will find yourself back at your starting-point.

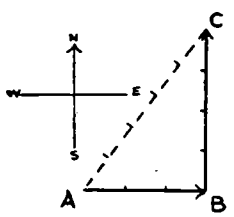


* Since a vector may be represented by a line, any line of the same length and direction parallel to it will represent the same vector.

† It is convenient to use the signs (+) and (−) when the vectors are oppositely directed along the same straight line, though whether we call E. or W. (+) does not matter in the least if, once we have decided, we stick to our decision.

‡ In drawing our vectors we shall place the arrow at one end of the vector line and call this the head of the vector.

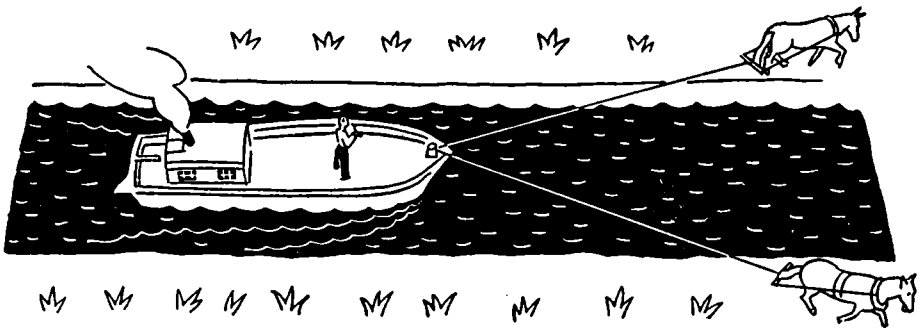
** If the angle between the two vectors is a right angle, you can easily calculate by Pythagoras' theorem what the length of the resultant is. Thus, if you take 3 steps E. and 4 steps N., you land at a point C, 5 steps from your starting-point, and in a direction approximately NE.:



$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 = 3^2 + 4^2 = 9 + 16 = 25$$

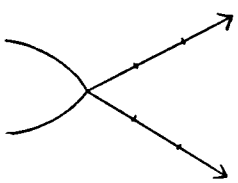
$$AC = \sqrt{25} = 5$$

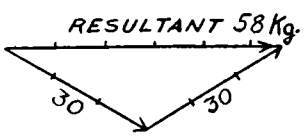
THIS METHOD of adding vectors is applicable to any vectors, such as force, velocity, etc. We shall now use the method for adding *forces*. Suppose



a boat is being pulled along a canal by two mules on opposite banks, each of which tugs on a rope attached to the boat. If the mules both exert the same pull, a pull of 30 kg. wt., and always remain exactly opposite each other on the two banks, find the force propelling the boat forward. Let the length of each rope be 20 m. and the width of the canal be 10 m.

The problem is to find the resultant of two forces of 30 kg. wt. each acting at a given angle with one another. We shall draw the vector triangle after deciding on some scale for drawing it. This is quite arbitrary, and we shall let 1 cm. in the diagram represent a force of 10 kg. wt. We draw two lines, 3 cm. long, placed at the angle given in the problem. The resultant is found to be a line 5.8 cm. long which represents a force of 58 kg. wt., and its direction is found to be along the middle of the river.

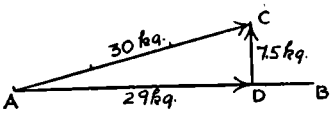




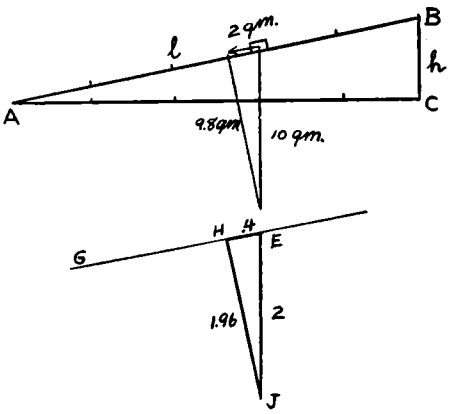
IT IS frequently convenient to do the reverse of this process of adding or compounding two forces into a single one, i.e., to *resolve* or break up a single force into two component forces. (These two components may be at any angle to each other, but we shall limit ourselves to the most widely useful case of two components at right angles to each other.) We shall take the force exerted by one of the mules in the last example and resolve it into two directions, one along the middle of the canal and one at right angles to the canal.

Let the line AB represent the direction along the middle of the canal. From A draw a line AC 3 cm. long, in the correct direction to represent the pull on the rope by one of the mules. Drop a perpendicular CD from C onto

AB. Then the two lines, AD and DC, represent, in magnitude and direction, the two forces, which are together equivalent to the single force represented by the line AC. The length of AD, which is 2.9 cm., represents a force of 29 kg. wt. The length of DC is 0.75 cm. and represents a force of 7.5 kg. wt. If only one mule were pulling on the boat, we could say that its pull was equivalent to a pull of 29 kg. wt. along the middle of the canal and a pull of 7.5 kg. wt. at right angles to this toward the bank. With two mules pulling, the forces toward the banks balance each other, while the forces along the middle of the river help each other, so that the total force pulling the boat down the canal is 2×29 kg., i.e., 58 kg. wt.



Let us apply this method for the resolving of forces into their components to an object on a smooth, inclined plane. A 10 gm. wt. is placed on a smooth plane having a length of 5 units and height of 1 unit. What force is pulling the object down the plane? With what acceleration does it slide down? The slope of the plane is represented in the diagram. AB is five times as long as BC.



To find the force pulling the 10 gm. wt. down the plane, we must resolve the vertical force of 10 gm. wt. into two forces, one along the plane and the other at right angles to the plane. In drawing the triangle of forces, we shall let 1 cm. represent 5 gm. wt. We shall draw a vertical line, EJ, 2 cm. long and then draw a line from J at right angles to a line GE. GE is parallel to the slope of the plane. Then the vector EH (length 0.4 cm.) represents the force pulling the object down the plane and HJ (length 1.96 cm.) represents the force with which the object pushes on the plane. Thus a force of 2 gm. wt. pulling the object down the plane and another force of 9.8 gm. wt. at right angles to this force are found to be together equivalent to the single vertical force of 10 gm. wt. Does the plane push on the object so that it balances the force of 9.8 gm. wt. at right angles to the plane?§ If the plane is *smooth*, the force pulling the object down the plane is this force of 2 gm. wt. The 10 gm. wt. will have an acceleration down the plane whose value may be found from Newton's Second Law, $F = ma$.

Force down the plane = 2 gm. wt. = 2×980 dynes

Mass of object = 10 gm. mass (weight = 10 gm. wt.)

$$F = ma \qquad \therefore \quad a = \frac{F}{m} = \frac{2 \times 980}{10} = 196 \text{ cm. per sec.}^2$$

From the manner in which the triangle of forces EJH is constructed, we can see that the angle EJH equals the angle BAC, i.e., the angle of slope of the plane. The triangles BAC and EJH are therefore similar, so that

$$\frac{BC}{AB} = \frac{HE}{EJ} \text{ or } \frac{\text{Height of plane } h}{\text{Length of slope } l} = \frac{\text{Force pulling object down plane}}{\text{Weight of object}}$$

$$\therefore \text{ FORCE PULLING OBJECT DOWN PLANE} = W \times \frac{h}{l} \text{ gm. wt.}$$

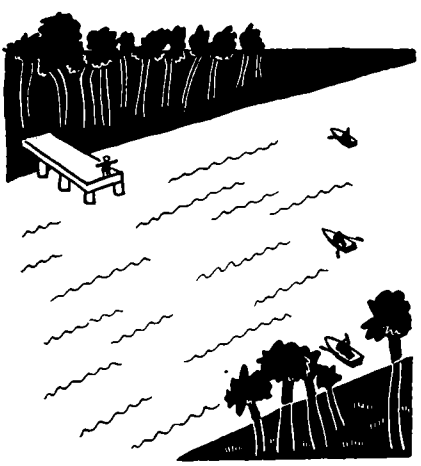
In our example $h = 1$ and $l = 5$, so that the force pulling the object down the plane is $\frac{1}{5} \times 10$ or 2 gm. wt.

We may now find the acceleration of the object down the plane. The force pulling the object down the inclined plane is $mg \times h/l$ dynes. Hence the acceleration down the plane is

$$a = \frac{F}{m} = \frac{mgh}{ml} = \frac{gh}{l} = \frac{980 \times 1}{5} = 196 \text{ cm. per sec.}^2$$

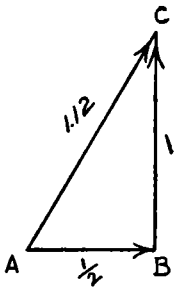
Since the height h of a plane is smaller than the length of the inclined plane l (unless the plane is vertical), the acceleration down the plane must be less than g, i.e., less than 980 cm. per sec.^{2}. Thus in a sense we "dilute" the effect of gravity by using a smooth, inclined plane.

WE SHALL now use an example where *velocities* are added. If you have ever tried to row directly across a river which is flowing fairly rapidly, you know that if you start out for a point directly opposite on the far bank you arrive some distance downstream. Suppose that the river is flowing at the rate of 1 MPH and that you row with a velocity of 2 MPH toward your friend, who is directly opposite on the far bank. With what velocity do you actually cross the river and how far downstream do you land if the width of the river is $\frac{1}{2}$ mi.?



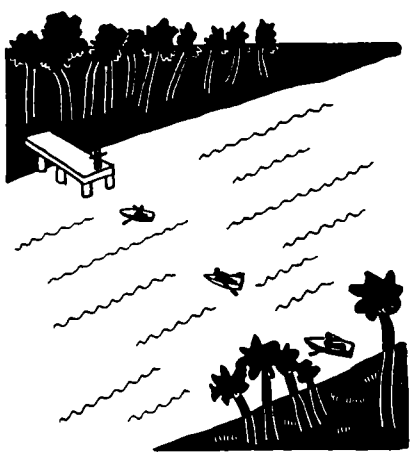
§ Answer to question on the previous page: Yes.

We have to compound two velocities: the velocity of your boat (2 MPH) and the velocity of the river (1 MPH) at right angles to each other. Let the scale for our vector triangle be $\frac{1}{2}$ in. to every MPH. The resultant AC will be 1.12 in. This represents a velocity of 2.24 MPH. Now even though your boat moves with this velocity of 2.24 MPH, you will not arrive at the opposite bank sooner, for you have farther to go. The increase in distance just makes up for the increase in velocity, and you arrive at the far bank in exactly the same number of minutes as you would have if the water had been still. This is due to the fact that the water is moving *at right angles* to the direction in which you are rowing. Thus, your two velocities at right angles may be treated independently—we shall return to this independence of velocities later.



Had the water been still, it would have taken you $\frac{1}{4}$ hr. to cross provided you rowed at the rate of 2 MPH the distance of $\frac{1}{2}$ mi. But during this $\frac{1}{4}$ hr. the current flowing at 1 MPH carries you downstream a distance of $\frac{1}{4}$ mi. Hence you land $\frac{1}{4}$ mi. downstream from your friend.

YOU MIGHT want to know in what direction upstream you should row in order to meet your friend, who is at a point directly opposite from your starting-point. In this case the resultant velocity, your rowing velocity plus the river velocity, must be in a direction directly across the river.



From the vector diagram you find that your resultant velocity, directly across the stream, is 1.73 MPH. Since the river is $\frac{1}{2}$ mi. wide, the time taken for you to cross is

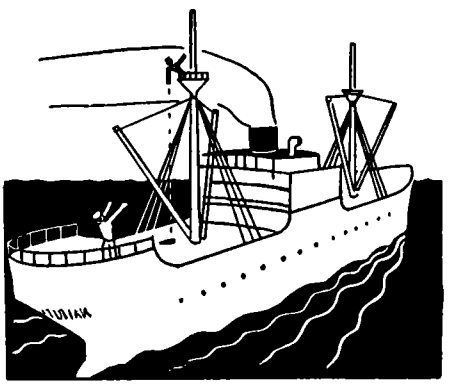
$$t=\frac{s}{v}=\frac{1}{2}\times\frac{1}{1.73}=0.29 \text{ hr.}=17.4 \text{ min.}$$

During this time, 0.29 hr., the river flowing at 1 MPH will have carried you a distance of 0.29 mi. downstream. So, in order to arrive where your friend is, your boat must be directed toward a point 0.29 mi. upstream.

PROJECTILES

IF WHEN you are at sea you climb the mast of the ship and drop something, you are probably surprised to find that the object falls vertically, and not aft of the mast, as you might have expected. During the time the object

is falling the ship is moving forward, but so is the object, at exactly the same rate. When it starts to fall, the object has the horizontal velocity of the ship, and this remains *constant* while its vertical velocity increases, due to gravity. Thus we may think of the object as having two independent velocities, one *horizontal and constant* and the other *vertical and continually increasing.*

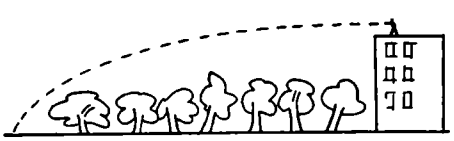


But why does the horizontal velocity of the object remain constant? The answer comes from Newton's Second Law: $F = ma$. The force of gravity acts vertically so that it can have no horizontal component. Thus we may say there is no horizontal force* acting on the falling object and hence (by Newton's Second Law) it can have no acceleration (or change of velocity) in a horizontal direction. When the object starts to fall, it has the horizontal velocity of this ship and it keeps this as it falls toward the deck.

This means that if we drop an object from a tall building and at the same instant shoot another object out horizontally, the two objects strike the ground simultaneously.

AFTER ALL these struggles with Newton's laws and with vectors, we can now solve some of the mysteries about bullets shot into the air: how high they rise, how far they go, and how long they are in the air.

EXAMPLE. Let us consider the time and place a bullet strikes the ground when fired horizontally with a velocity of 10,000 cm. per sec. from a building 100 m. high. If we know the time the bullet takes to strike the ground, we can easily find the distance of the spot where the bullet lands from the base of the building since the horizontal velocity of the bullet is constant. This time is the time the bullet takes to fall vertically a distance of 100 m.



Time bullet is in the air: $s = \frac{1}{2}at^2$, whence $t^2 = \frac{2s}{a}$

$s = 100$ m. $= 10{,}000$ cm.; $a = g = 980$ cm. per sec.^{2}

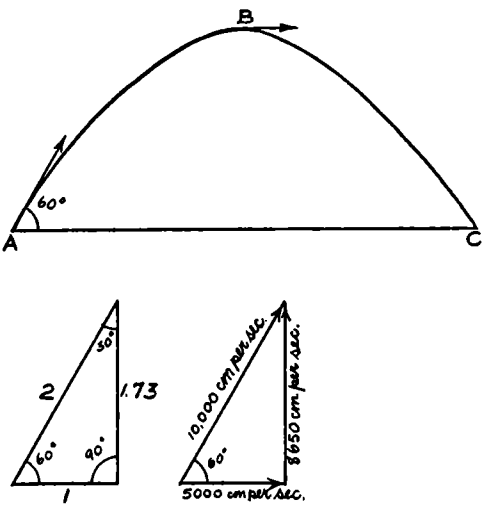
$$\therefore \quad t^2 = \frac{2s}{a} = \frac{2 \times 10{,}000}{980} = 20.4$$

$t = \sqrt{20.4} = 4.5$ sec. approximately

* There is of course a resisting force due to the air but this we must neglect as the mathematical complexities of calculating it are too great. We shall have to use this simplification in all our problems.

Distance traveled horizontally: Since the horizontal velocity of the bullet, 10,000 cm. per sec., remains constant during this 4.5 sec., the bullet will have traveled horizontally a distance

$$s = vt = 10{,}000 \times 4.5 = 45{,}000 \text{ cm.} = 0.45 \text{ km.}$$



EXAMPLE. If a bullet is fired from a gun which is inclined at any angle to the horizontal, the horizontal component of its velocity remains unchanged but the vertical component of its velocity diminishes under gravity to zero at the top of its flight, then increases under gravity on its downward flight.

Suppose a bullet is fired with a velocity of 10,000 cm. per sec. from a rifle inclined at 60° to the horizontal. Find where the bullet hits and how long it is in the air.

We must resolve the velocity of 10,000 cm. per sec. into its horizontal and vertical components. From the vector triangle of velocities the initial horizontal velocity of the bullet is found to be 5,000 cm. per sec. and the initial vertical velocity 8,650 cm. per sec. At B, the top of its flight, its vertical velocity is zero while its horizontal velocity is still 5,000 cm. per sec. The time taken for the bullet to get to the point B will be the time that a bullet shot vertically upward with a velocity of 8,650 cm. per sec. takes to come to rest. We have already done a problem of this kind (p. 8).

Time the bullet is in the air:

Initial velocity $v_i = 8{,}650$ cm. per sec.

Final velocity $v_f = 0$ cm. per sec.

Acceleration $a = -980$ cm. per sec.^{2}

$$a = \frac{v_f - v_i}{t}; \qquad t = \frac{v_f - v_i}{a} = \frac{0 - 8{,}650}{-980} = 8.8 \text{ sec.}$$

The time which the bullet takes to fall to the ground from B will also be 8.8 sec., so that the total time the bullet is in the air will be 2×8.8, or 17.6 sec.

The range of the bullet, i.e., the horizontal distance AC traversed by the bullet traveling with a constant horizontal velocity of 5,000 cm. per sec. for 17.6 sec., is given by

$$s = vt = 5{,}000 \times 17.6 = 88{,}000 \text{ cm.} = 880 \text{ m.}$$

PROBLEMS

1. You take a very short walk, 12 steps E. and 5 steps N. How far are you from your starting-point, and roughly in what direction? Represent the short walk on a vector diagram. You decide on a much more extended walk and go 2 steps E., 4 NE., 5 N., 3 NW., and 3 W.
   a) Find your resultant displacement by drawing the vector polygon.
   b) Would you have got to the same spot had you taken the steps in a different order: 3 W., 3 NW., 5 N., 4 NE., 2 E.? (Use graph paper; it's much easier.)

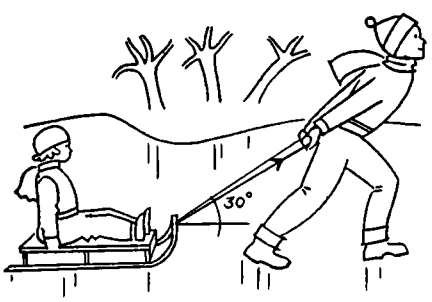
2. A north wind is blowing at 15 MPH and a ship is sailing due E. at 15 MPH. In what direction does a flag flying from the masthead point?

3. When astronomers measure the velocity of a star or a nebula, do they measure the actual velocity of the star or the vector sum of the velocity of the star and the velocity of the earth?

4. An airplane traveling at 100 MPH due W. in calm air runs into a current of air blowing horizontally due S. If the velocity of the air is 20 MPH and the controls of the airplane are not changed, with what velocity relative to the moving air does the airplane travel? Draw a vector diagram to show how fast and in what direction the airplane now travels with respect to the ground.

5. A sailor drops his telescope from the crow's nest at the top of the mast when the ship is slowing down. Will the telescope strike the deck at the foot of, fore, or aft of the mast?

6. You take your small sister out for a ride in her sled, going over rough ice with a constant velocity. If the rope attached to the sled makes an angle of 30° with the horizontal and you pull with a force of 1,000 gm. wt.,

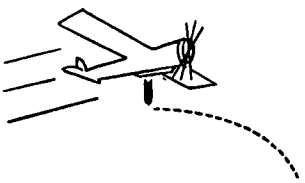


   a) Find the horizontal component of your pull.
   b) Find the force of friction between the sled and the rough ground.
   c) If you should now start to run, would you have to pull with a greater force than previously? Why? (The frictional force is assumed to remain constant. The sides of a 30°, 60°, and 90° triangle are in the ratio of 1 : 1.73 : 2.)

7. A bullet is shot horizontally with a velocity of 800 ft. per sec.
   a) How far does the bullet fall below the horizontal line of fire during the first second?
   b) What horizontal distance does the bullet travel in the first second?

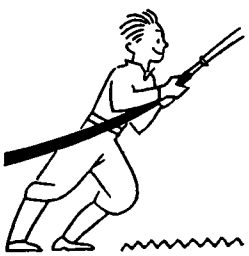
8. A bullet is shot horizontally in the air from the top of a tall building and at the instant the bullet leaves the barrel another bullet is dropped vertically. Will the two bullets hit the ground at the same time?

9. An enemy airplane, flying with a velocity of 120 MPH (176 ft. per sec.)

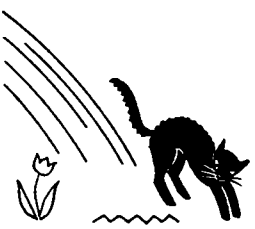


at an altitude of 6,400 ft., drops a bomb when over the city hall. How far from the city hall does the bomb strike the ground?

10. A bullet is shot with a velocity of 70,000 cm. per sec. from a gun inclined at 45° to the horizontal. Find
    a) The horizontal and vertical components of the velocity of the bullet as it leaves the gun.
    b) The time the bullet takes to rise to its maximum height.
    c) The total time the bullet is in the air.
    d) The range, i.e., horizontal distance, the bullet travels. (In order to get the maximum range the gun must be pointed at 45° to the horizontal. The sides of a 45°, 45°, and 90° triangle are approximately 0.7:0.7:1.)



11. When watering the lawn you find that the maximum horizontal distance you can send the water is 49 ft. Find the velocity of the water as it comes from the hose.

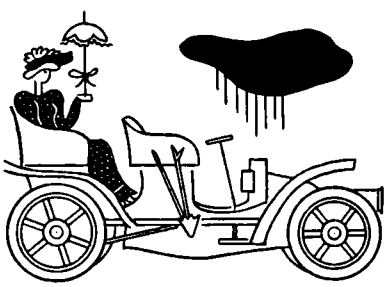


12. You slide down an icy hill having an inclined length of 2,560 ft. and a height of 256 ft. (Assume you can neglect friction.)
    a) What is your acceleration down the hill?
    b) How long does it take you to go down the hill?
    c) What is your velocity at the bottom of the hill?
    d) Find the velocity that an object would acquire in falling a vertical height of 256 ft.
13. From the data of problem 12 draw a graph representing your velocity down the hill plotted against time, and one representing the distance moved against time.
14. A barge is pulled along a canal by a horse which exerts a force of 100 kg. wt. If the barge is kept 5 m. from the bank of the canal and the length of the rope attaching the barge to the horse is 20 m., find
    a) The force pulling the barge along the canal.
    b) The force which must be exerted by the rudder to keep the barge 5 m. from the bank.

15. Here is a geological problem: Streams carry particles of fine sediment such as silt and mud. These fine particles are heavier than water. If in a stream the whole mass of water moves at a uniform rate, is it true that a particle settling from the top will reach the bottom in the same time that it would if the water were stagnant? (In an actual stream, particles do not settle in this manner. The water in the stream does not move as a whole. The upper layers tend to move faster than the lower ones and eddies may be set up by irregularities of the bed of the stream.)

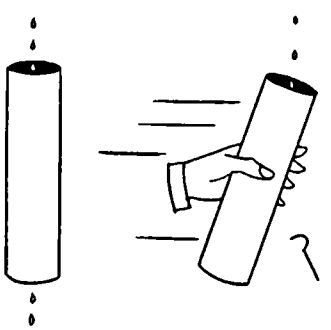
16. You are traveling in a train going at 60 MPH and overtake a train on the next track traveling at 40 MPH.
    a) With what speed and in what direction (forward or backward) does the second train appear to be moving?
    b) In making this conclusion are you considering yourself as moving forward at 60 MPH or as at rest?

17. One day in April, when the rain is falling vertically, the lady in the drawing prepares for an automobile ride.



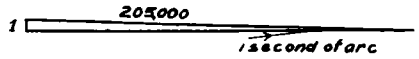
    a) If and when the automobile gets started, where should the lady hold the umbrella best to protect her hat? (Slightly in front, behind, or directly over her head.)
    b) From which direction (roughly speaking) does the rain appear to her to be falling?
    c) Does this explain why in a modern automobile traveling at high speed the rain and snow strike the windshield and not the rear window?

18. A raindrop, falling vertically with a constant velocity of 6 m. per sec., enters a long cylindrical tube. If the tube is moving horizontally forward with a velocity of 6 m. per sec., at what angle to the horizontal must the tube be sloped for the raindrop entering the top of the cylinder to continue falling down the axis of the tube?



19. In 1728 Rev. James Bradley, later astronomer royal at Greenwich, set up a long telescope with its axis vertical. He found that as the earth went around the sun all the stars appeared to move by the same amount irrespective of their distances from the earth. This effect he called *aber-*

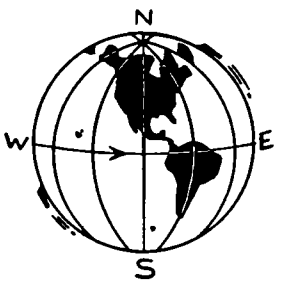
ration. It can be explained in a similar manner to that in which the situation discussed in problem 18 was explained except that here light, not a raindrop, enters a tube. The maximum angle which the telescope had to be moved from the vertical was 20.5 sec. of arc.



1 sec. of arc = 1 part in 205,000 (approx.)

If the speed of the earth around the sun is 30 km. per sec., find the speed of light in km. per sec.

20. A long-range gun in the Northern Hemisphere shoots a projectile into the air due *S*.



a) When the bullet starts does it have an eastward speed due to the eastward rotation of the earth?

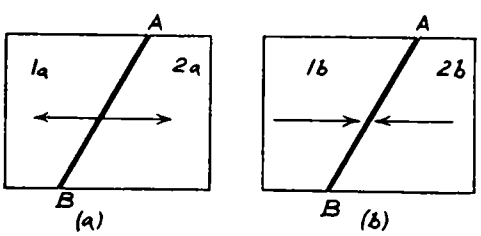
b) Does the bullet keep this eastward speed (apart from air friction) after it leaves the gun?

c) Does an object on the earth, as at Chicago 42° N., have the same, a greater, or a less eastward speed than an object at the equator, due to the eastward rotation of the earth?

d) Will the projectile appear to move E. or W. of the north-south meridian due to the eastward rotation of the earth?

e) What direction from this meridian will a projectile appear to move if shot due S. in the Southern Hemisphere?

21. Another geological problem: The two diagrams represent sections of rock in the earth which are split or faulted along the line AB. Fault a is called a normal fault. There is a tension tending to separate the two blocks. As a result of this tension there will be slipping along AB.

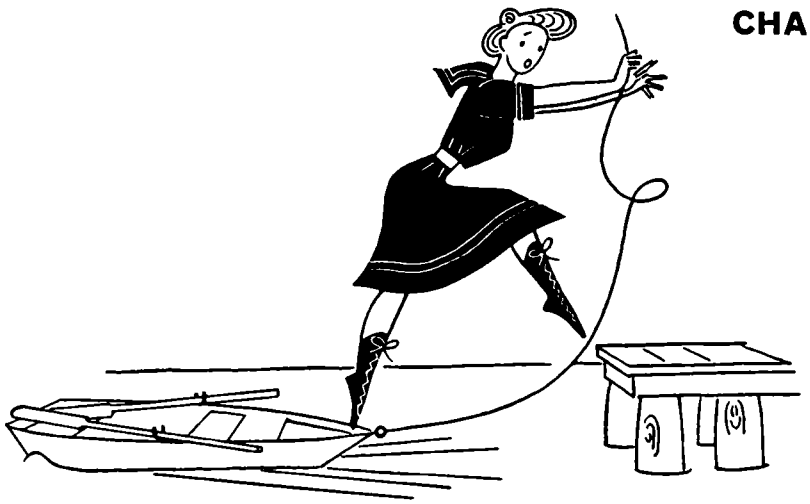


a) Will 1*a* go up or down with respect to 2*a*?

Fault *b* is called a thrust fault. There is a compression or thrust squeezing the rocks together. As a result of this thrust fault there will be slipping along AB.

b) Will 1*b* go up or down with respect to 2*b*?

CHAPTER 4



NEWTON'S THIRD LAW OF MOTION

All things, by immortal power,
Near or far,
Hiddenly
To each other linkéd are,
That thou canst not stir a flower
Without troubling of a star.—FRANCIS THOMPSON.

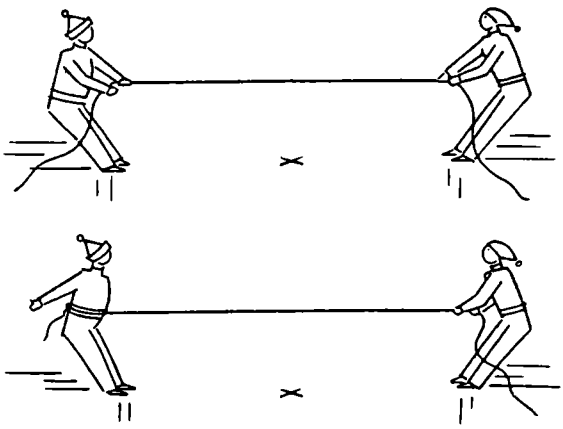
YOU ARE probably sitting on a chair while reading this book and are exerting a force on the floor. Then why don't you fall through the floor? Why, the floor exerts a force on you which exactly counterbalances the force you exert on the floor. (*a*) How do you know the two forces balance? (*b*) What would happen if you push on the floor with a greater force than the floor pushes on you?§ Force is dual in its nature. This is the essence of **Newton's Third Law of Motion:**

For every action (force) there is an equal and opposite reaction: or, the mutual actions of any two bodies are always equal and oppositely directed.

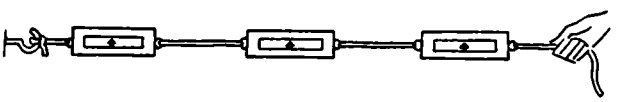
BUT FORCES can produce accelerations. It is in such cases that Newton's Third Law is of most interest. Suppose you and a friend start a tug-of-war on ice. You would find that neither would win if the ice were smooth. You would simultaneously approach each other. (Of course if the ice were perfectly smooth you could not walk around, and even standing still would have its difficulties. We won't bother about this.)

Next time you try a tug-of-war on ice, pull on the rope with an even pull, and as you move toward each other take in the rope so that there is always

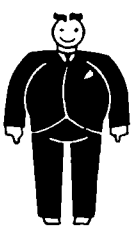
the same tension in the rope. Notice the spot where you bump each other. Go back to your starting-points and this time let your friend tie the rope around him and you do the pulling alone. You may be surprised to find that you meet at exactly the same place as previously. If you are not satisfied with the result, start again; but this time tie the rope around your waist and let your friend do the pulling. You again meet at the same spot. Where this spot is depends only on the relative masses of you and your friend.



We had the tug-of-war on the ice so that we might neglect any frictional forces between your feet and the ice. Hence the only horizontal force is the one in the rope. The tension or force in the rope must be the same all along its length. (This you can check by cutting the rope in several places and joining up the ends with spring balances. When the rope is taut the spring balances will all give the same reading.) No matter who does the pulling, the force exerted by your friend on you is the same as the force you exert on your friend. Since you are pulled together, these forces must be in opposite directions.



The acceleration which each acquires will depend on your respective masses. If you both have the same mass, you will each acquire the same acceleration, while if you have twice the mass of (are twice as heavy as) your friend, you will acquire only half the acceleration he does.



You

YOU	YOUR FRIEND
Large mass, M	Small mass, m
Small acceleration, a	Large acceleration, A
Force on you, F_1	Force on him, F_2
Your velocity after 1 sec., v	His velocity after 1 sec., V

$F_1 = -F_2$ (Newton's Third Law)*

$F_1 = Ma$ $\qquad$ $F_2 = mA$

Your friend

§ Answer to questions on the previous page: (*a*) You stay put. (*b*) The floor would cave in.

* The minus sign is to show that the forces are in opposite directions.

Forces are measured by the acceleration they produce in a given mass.

$$\therefore \quad Ma = -mA \qquad \text{or} \qquad \frac{a}{A} = -\frac{m}{M}$$

But acceleration is the rate of change of velocity:

$$\text{Acceleration} = \frac{v_c}{t} = \frac{v_f - v_i}{t}$$

Since you were both stationary before you started pulling, your initial velocity, v_i, was equal to zero. Hence $v_c = v_f$, where v_f is your velocity after some time t. We will let $t = 1$ sec. Then your velocity after pulling for this second will be numerically equal to your acceleration, and the same holds for your friend, i.e., $v = a$ and $V = A$.

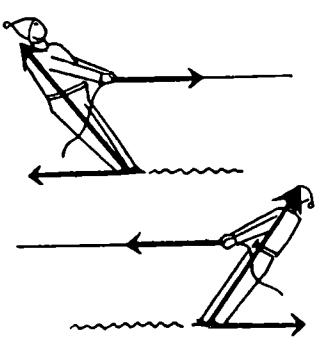
$$\therefore \quad F_1 = Mv \qquad F_2 = mV$$

or

$$Mv = -mV$$

The product of mass and velocity, Mv, is called **Momentum.** Like force and velocity, momentum is a *vector* quantity. We have just proved that the momentum which you acquire through the tug-of-war is *exactly equal but opposite* to that which your friend acquires. Stated another way, the vector sum of your combined momenta ($mV + Mv$) is equal to zero. This is true all the time you are pulling and coming together.

YOU ARE probably strenuously objecting to all this, since tugs-of-war are not held on smooth ice but on rough ground. The problem then becomes more complicated. You can now get a grip on the earth. You push on the earth and the earth pushes back on you with an equal but opposite force (Newton's Third Law). The pull which formerly caused you to move is now balanced by the horizontal component of the backward push of the earth. You are able to pull your friend from his stance not because there is more tension in the rope at your end than at his, but because you have a better grip on the earth; the earth pushes you back more than it pushes him back. Once motion is started, we can bring in our momentum considerations; but then the earth has to be taken into account, and this is too complicated for



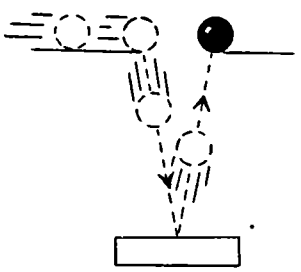
us here. Nevertheless the total momentum change of the system, men plus earth, is zero. Every time you walk you push back on the earth, but if you are on smooth ice, where there is no friction, you cannot push back and the ice cannot push you forward; you slip and slide around.

You slip and slide around

If a railway engine is placed on tracks which are not firmly tied to the ground but can slip, then as the train goes forward the tracks move backward in such a way that the vector sum of the momenta of train and tracks is zero. (Of course tracks are firmly fastened to the ground so that when the train starts up and increases its momentum the momentum of the earth is changed by the same amount.) Will this change the velocity of the earth very much?§

Whenever one object acquires momentum another object (or objects) always receives the same momentum in the opposite direction so that the vector sum of the momentum change is zero. That is to say, momentum is conserved. None is ever lost or gained. This is a consequence of Newton's laws of motion and is sufficiently important to be itself called a law—the **Law of Conservation of Momentum.**

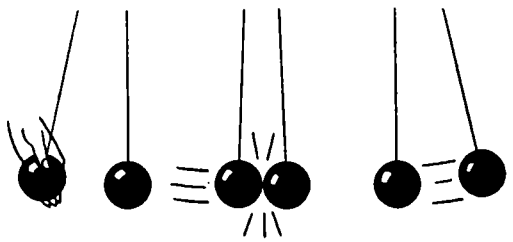
YOU KNOW that when some objects collide they stick together while others rebound, but, in either case, the Law of Conservation of Momentum holds; the total momentum of the objects is the same after collision as before collision. When two objects collide and stick together, as would happen with two pieces of plasticene, the collision is called *inelastic.* On the other hand, when a hardened steel ball falls on a hardened steel surface it rebounds with the same velocity it had when it struck and consequently rebounds to the height from which it fell. This is a perfectly *elastic* collision. (Of course in practice many collisions are neither completely inelastic nor perfectly elastic, but these would involve us in too many complexities, so we shall limit our discussion to the two extreme cases.)



A perfectly elastic rebound

YOU ARE probably amused and maybe curious when you watch the collision of two similar steel balls both suspended freely by strings of the same

length. One ball is displaced and allowed to strike the other. After collision the two exchange velocities; first the one swings out and then the other. We shall now show why this is so.



Let each of the two balls have a mass m and let u be the velocity with which ball A strikes ball B (at rest). After collision let ball A have a velocity v_1 and ball B a velocity v_2 in the same direction. By the *Law of Conservation of Momentum*, the momentum before collision equals the momentum after collision.

Before collision: Momentum of ball $A = mu$. Momentum of ball $B = 0$

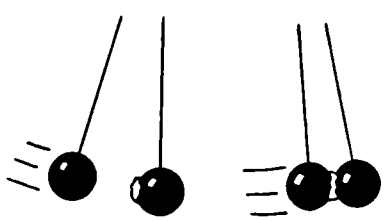
After collision: Momentum of ball $A = mv_1$. Momentum of ball $B = mv_2$

Conservation of momentum: $mu = mv_1 + mv_2$ or $u = v_1 + v_2$

Since the balls are steel and make an elastic collision, the velocity with which they approach each other $(u - 0)$ equals the velocity with which they separate $(v_2 - v_1)$.

$$\therefore \quad u = v_2 - v_1$$

This looks peculiar. In one case $u = v_1 + v_2$, the sum of the two velocities, and in the other $u = v_2 - v_1$, the difference of the two velocities. No matter how hard you try, you will find there is only one way in which this peculiarity may be accounted for. That is if $v_1 = 0$. The two equations then reduce to $u = v_2$. Hence after the collision the moving ball, A, comes to rest, and ball B, originally at rest, moves off with the velocity that ball A had. This exchange of velocities at impact will happen at every collision.

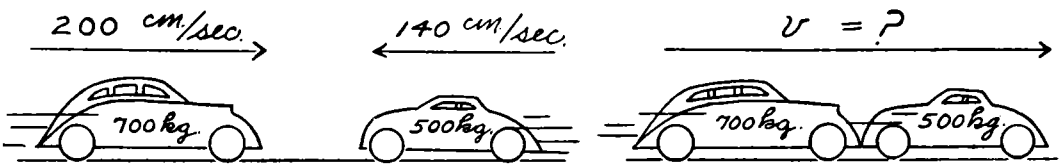


If you should make the collision inelastic by sticking a piece of plasticene on one of the balls, the two will stick on collision and move off together. Show that the velocity with which they move off is half the velocity with which ball A struck ball B.

LET US see what happens, from the point of view of physics, when two automobiles have a head-on collision. An automobile with a mass of 700 kg. and a velocity of 200 cm. per sec. has a head-on collision with another auto-

§ Answer to question on previous page: No.

mobile of 500 kg. mass and 140 cm. per sec. velocity. Find the velocity with which they both move immediately after collision.



Suppose the automobiles move with a velocity of v cm. per sec. after collision in the same direction that the 700-kg. car was traveling.

Momentum of 700-kg. car before collision $= +700 \times 200$ kg. cm. per sec.*
Momentum of 500-kg. car before collision $= -500 \times 140$ kg. cm. per sec.
Total momentum after collision $= 700v + 500v$

By the *Law of Conservation of Momentum:*

Total momentum before collision = total momentum after collision

$$(700 \times 200) - (500 \times 140) = 700v + 500v$$

$$140{,}000 - 70{,}000 = 1{,}200v$$

$$\therefore \quad v = +\frac{70{,}000}{1{,}200} = +58.3 \text{ cm. per sec.}$$

The cars then move off together to the right (see diagram), with a velocity of 58.3 cm. per sec. Of course they would probably soon come to rest because of various resisting forces developed during the accident.

Quite a jolt!

A SOMEWHAT DIFFERENT problem from that of the collision of two objects, but one which can also be analyzed by means of the Law of Conservation of Momentum, is that of a bullet fired from a gun. When you fired a rifle for the first time, you may not have held the rifle firmly to your shoulder, with the result that when you fired, the rifle recoiled and gave your shoulder quite a jolt. Before firing, the bullet is at rest in the barrel of the rifle; the total momentum of bullet and rifle is zero. When the bullet leaves the barrel, it has momentum in a forward direction. The Law of Conservation of Momentum tells us that the rifle must have momentum too; it must be

* Momentum is a vector having the same direction as the velocity of the object. We shall here adopt the convention that momenta and velocities to the right are positive and to the left negative.

exactly equal and opposite to the momentum of the bullet. Their combined momenta are zero.

EXAMPLE: A rifle having a mass of 4 kg. fires a bullet of 4 gm. mass with a velocity of 10,000 cm. per sec.

Find the momentum of the bullet as it leaves the rifle.

Find the velocity of recoil of the rifle.

If the bullet takes $\frac{1}{100}$ sec. to travel down the barrel, find the acceleration of the bullet in the barrel. Find the force which is being exerted on the bullet; also the force exerted by the rifle on the shoulder.

$$\text{Momentum of bullet: } mv = 4 \times 10{,}000 = 40{,}000 \text{ gm. cm. per sec.}$$

By the Law of Conservation of Momentum, the momentum of the bullet + the momentum of the rifle equals zero, or

$$\text{Momentum of rifle} = -\text{momentum of bullet} = -40{,}000 \text{ gm. cm. per sec.}$$

$$-40{,}000 = \text{mass of rifle} \times \text{velocity of recoil of rifle}$$

$$\text{Velocity of recoil of rifle} = \frac{-40{,}000}{4{,}000} = -10 \text{ cm. per sec.}$$

(The negative sign indicates that the velocity of recoil of the rifle is in the opposite direction from the velocity of the bullet.) Acceleration of bullet down barrel of rifle:

$$a = \frac{v_c}{t} = \frac{10{,}000}{\frac{1}{100}} = 100 \times 10{,}000 = 10^6 \text{ cm. per sec.}^2$$

$$\text{Force on bullet: } ma = 4 \times 10^6 = 4{,}000{,}000 \text{ dynes}$$

To find force exerted by bullet on rifle forcing gun backward:

$$\text{Acceleration of rifle } a = \frac{v_c}{t} = \frac{-10}{\frac{1}{100}} = -1{,}000 \text{ cm. per sec.}^2$$

$$\therefore \text{Force} = ma = -4{,}000 \times 1{,}000 = -4{,}000{,}000 = -4 \times 10^6 \text{ dynes}$$

This result, namely, that the force of the rifle on the bullet is equal to the force of the bullet on the rifle, could have been predicted directly from Newton's Third Law.

We have used $F = ma$ as an expression of Newton's Second Law, but since $a = \frac{v_c}{t}$, we might have written $F = \frac{m \times v_c}{t}$, which shows that force is measured by the time rate of change of the momentum which it produces in an object. The momentum of an object can only be changed by a force acting on the object. But forces are dual in nature. According to Newton's Third Law, for every force there is an equal and opposite force or reaction on some other body. Therefore, whenever the momentum of one object is changed, the momentum of some other object must be changed by an equal and opposite amount.

As a result of the fact that the bullet acquires a momentum of 40,000 gm.

cm. per sec., the rifle has also acquired this momentum. If you had held the rifle firmly to your shoulder, this momentum would be given to the rifle and the upper part of your body. These two combined have a much greater mass than the rifle alone. Hence the velocity of recoil of your shoulder plus the rifle is small and no jar is felt when the bullet is fired. Your shoulder still has to exert the force of 4,000,000 dynes while the bullet is going down the barrel, but this is a steady force and is not noticeable if you brace yourself. On the contrary, if the rifle is loosely held so that it acquires its full recoil velocity of 10 cm. per sec. before striking your shoulder, you get a jolt.

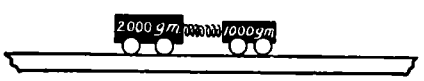
PROBLEMS

1. You are standing on a long board placed on perfectly smooth ice. You run forward on the board; what happens to the board? Can you pass a friend who is standing alongside you on the ice
   a) If you have the same mass as the board?
   b) If you have a greater mass than the board?
   c) If you have lesser mass than the board?
   (Remember the combined momentum of you and the board must remain zero.)

2. You somehow got onto a sheet of perfectly smooth ice, and there is no one around to help you to get off. How could you get off?

3. A sailboat comes to rest in a dead calm in the middle of a lake. Is it possible for the occupants to get the boat in motion by blowing on the sails? (We shall assume that the occupants are good "blowers.")

4. Two toy trains, one having a mass of 1,000 gm. and the other having a mass of 2,000 gm., have a coiled spring between them. The trains are pushed together so that the spring is compressed. When they are suddenly released, the 1,000-gm. train goes off with a velocity of 20 cm. per sec.



   a) With what velocity does the 2,000-gm. car move off?
   If the force of the compressed spring on the 1,000-gm. car is 10^6 dynes
   b) What is the force of the compressed spring on the 2,000-gm. car?

5. A bullet of 5 gm. mass is fired from a rifle of 5,000 gm. mass. If the bullet leaves the rifle with a velocity of 20,000 cm. per sec., find the velocity of recoil of the rifle.

6. Two boats having masses of 1,000 and 2,000 kg., respectively, are moving toward each other, each with a velocity of 600 cm. per sec. They collide head on. Find
   a) The total momentum of the two boats before collision.
   b) The total momentum of the boats after collision.
   If the boats stay together after the collision, find
   c) The velocity with which they start to move.

7. A pitcher throws a baseball whose mass is 150 gm. with a velocity of 20. m. per sec. The batter knocks the ball back toward the pitcher with a velocity of 50 m. per sec.
   a) What is the change in momentum of the ball on striking the bat?
   b) Of the bat when struck by the ball?
   If the ball is in contact with the bat for 0.02 sec., find
   c) The force which the bat exerts on the ball.
   d) The force which the ball exerts on the bat.

8. A shell explodes and sends shrapnel in all directions. Is the total momentum of the scattered shrapnel equal to the momentum of the shell before explosion?

9. You and your friend have a tug-of-war on smooth ice. If you pull on the rope with a force of 10,000 gm. wt.,
   a) With what force does your friend pull on the rope?
   You have a mass of 100 kg. and your friend has a mass of 150 kg.
   b) What acceleration do you acquire through this pulling?
   c) What acceleration does your friend acquire?
   What is
   d) Your velocity and momentum at the end of 2 sec. provided you start from rest?
   e) The velocity and momentum of your friend under the same conditions?

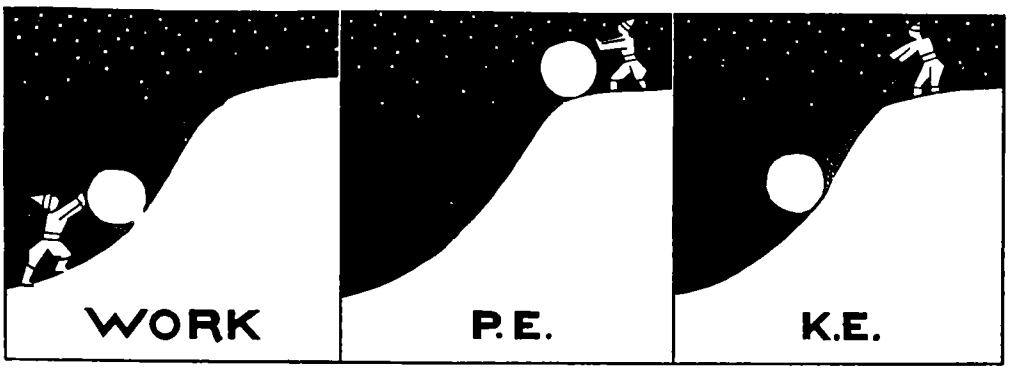
10. A bullet whose mass is 5 gm. is shot and becomes imbedded in a hanging piece of wood whose mass is 3,000 gm. If the velocity of the bullet is 10,000 cm. per sec.,
    a) With what velocity do the block and bullet start to move?
    What is
    b) The momentum of bullet and block before impact?
    c) After impact?

11. A bullet whose mass is 5 gm. is shot at a hardened steel plate and makes a perfectly elastic collision with it. If the velocity of the bullet is 10,000 cm. per sec. before striking the plate,
    a) With what velocity does it rebound from the plate?
    What is the change in momentum
    b) Of the bullet due to the collision?
    c) Of the steel plate?

12. Two hardened steel balls, one having a mass of 10 gm. and the other a mass of 30 gm., are suspended as pendulums by strings of equal lengths. The 10-gm. ball is drawn back and let go. If it acquires a velocity of 20 cm. per sec. by the time it strikes the 30-gm. ball hanging at rest, what are the velocities (in magnitude and direction) with which the two balls move immediately after collision? Assume that the collision is perfectly elastic.

13. When a ball falls toward the earth, does the earth fall toward the ball?

14. When a ball is thrown upward, does the earth receive
    a) An equal and opposite amount of momentum?
    b) An equal and opposite velocity?
    c) Does the earth move away from the ball?

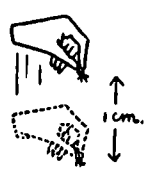


WORK AND ENERGY

THUS FAR we have been discussing Newton's laws and have seen that in order to change the velocity of an object a force is required, this force being measured by the rate of change of momentum of the object. The Third Law asserts that forces are dual in nature; whenever one object receives momentum, another object (or objects) must receive an equal and opposite momentum.

WHEN A FORCE acts on an object and sets it in motion, we say the force does *work* on the object. **Work is the product of the force and the distance moved in the direction of the force:** $W = F \times s$, where F and s are in the same direction. Work is done on an object whenever its velocity is changed, this work being done by the force used against the inertia of the object. Work can also be done against *friction*, *gravity*, *electrical* and *magnetic* forces, etc.

The unit of work is called the ERG. **The erg is the work done when a force of 1 dyne moves an object through a distance of 1 cm.** A dyne of force ($\frac{1}{980}$ gm. wt.) is approximately equal to the weight of a mosquito, so that if you lift a mosquito vertically through a distance of 1 cm. you do an erg of work. This is, as you know, a very small amount of work, so another unit 10,000,000 (10^7) times as great is used. This is called the JOULE. 1 joule $= 10^7$ ergs.



Sometimes you are interested not only in the amount of work which is done but also the rate at which it is done. The rate of working is called

POWER. The unit of POWER in the c.g.s. system is the WATT.* **One watt of power is expended when 1 joule (10^7 ergs) of work is done per second.**

$$\text{Watts} = \frac{\text{joules}}{\text{sec.}}, \qquad \text{i.e., joules} = \text{watts} \times \text{sec.}$$

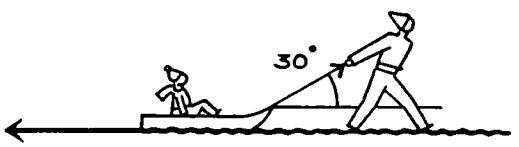
WORK DONE AGAINST FRICTIONAL FORCES

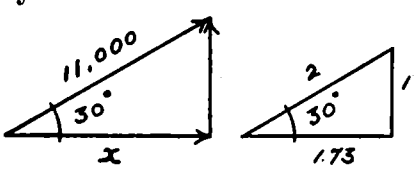
ONE DAY in winter when you are taking your small sister out for a ride on a sled you come to a place where the ground is bare and you have to pull a lot harder. In pulling the sled along you do work against friction; the force you exert has to counterbalance exactly the force of friction. (If the road is level, you do no work against gravity.) Suppose you lean over so that the rope is horizontal. Then if you pull on the rope with a force of 10,000 gm. wt. (980×10^4 dynes), the force of friction is also this amount. If you drag the sled a distance of 100 m. (10,000 cm.), the work you do against friction is

$$W = F \times s = 980 \times 10^4 \times 10^4 = 9.8 \times 10^{10} \text{ ergs}$$
$$= 9.8 \times 10^3 \text{ joules}$$

We can also measure the work in another unit, gm. wt.$\times$cm. The work ($F \times s$) is 10^4 gm. wt.$\times 10^4$ cm., i.e., 10^8 gm. wt.$\times$cm. These two are, of course, quite equivalent, since 1 gm. wt. = 980 dynes, but we shall adopt the more useful unit, namely, the *erg*.

NOW THE more usual method of pulling a sled is to have the rope inclined at an angle to the ground. In this case you must pull with a greater force than the force of friction because only the horizontal component of your force overcomes friction. Suppose you pull with a force of 11,000 gm. wt. and that the rope is inclined at 30° to the horizontal. We must resolve the force of 11,000 gm. wt. into its horizontal and vertical components by the vector method.





The lengths of the sides of a 30°, 60°, and 90° triangle are in the ratio of 1:1.73:2, so that the horizontal component is given by the proportion

$$11{,}000 : 2 = x : 1.73 ; \qquad x = \frac{11{,}000}{2} \times 1.73 = 9{,}515 \text{ gm. wt.}$$
$$= 9{,}515 \times 980 \text{ dynes}$$

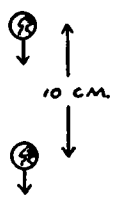
* In the British system the unit of power is the horse-power (h-p.); 1 h-p. = 746 watts.

In this case the force of friction is 9,515×980 dynes and the work done against friction when the sled is moved a distance of 10,000 cm. is 9,515×980 ×10,000 ergs, i.e.,

$$9.515\times0.98\times10^{10}\text{ ergs}=9.32\times10^{10}\text{ ergs}$$
$$=9.32\times10^{3}\text{ joules}$$

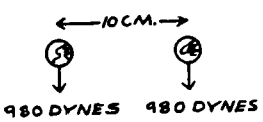
The vertical component of your force is 5,500×980 dynes. Does this force do any work? Can you tell what its effect will be?§ Of course if the sled were at rest at the beginning of the 100 m. pull, an extra amount of work would have to be used to set the sled in motion, but this would all be returned when the sled came to rest—the sled would go a little farther after you stopped pulling.

WORK AGAINST GRAVITY



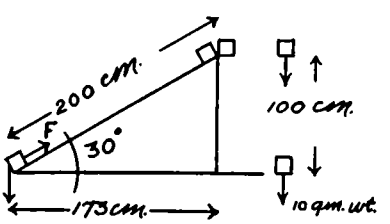
WHEN YOU hold a 1 gm. wt. in your hand, you have to exert an upward force of 980 dynes to prevent its falling to the earth. Then if you raise the 1 gm. wt. a vertical height of 10 cm., you do 9,800 ergs of work against gravity. If you raise a mass of m gm. a vertical height of h cm., you do mgh ergs of work against the force of gravity.

The force due to gravity always acts vertically, so that it is only when a body is raised some distance vertically that work is done against gravity.



Thus if you move the 1 gm. wt. 10 cm. in a horizontal direction, you do no work against gravity. Of course you may have to do work against the air resistance, which in this case will be a horizontal force, but this is NOT against gravity.

EXAMPLE. Let us find the work which must be done against gravity to pull a 10 gm. wt. up a smooth inclined plane 200 cm. in length and 100 cm. in height. The 10 gm. wt. is pulled up with a uniform velocity by means of a force F parallel to the slope. The force F must be exactly equal and opposite to the component of the 10 gm. wt. along the plane. We must then resolve the vertical force of 9,800 dynes (10 gm. wt.) into two forces, one along the plane and the other at right angles to the plane. You will recall from the section on inclined planes in the previous chapter that the force parallel to the plane required to move an object of mass m up a smooth plane of length l and height h is $mg\times\frac{h}{l}$.



$$\therefore\quad F=10\times980\times\tfrac{1}{2}\tfrac{0}{0}\tfrac{0}{0}=4,900\text{ dynes}$$

The work done against gravity in pulling the object up the plane a distance of 200 cm. is:

$$W = F \times s = 4,900 \times 200 = 980,000 = 9.8 \times 10^5 \text{ ergs}$$

The object is now 100 cm. higher vertically than it was at the bottom of the plane. It might be interesting to calculate how much work we must do against gravity to raise the object vertically this amount. The force we should have to use must be equal and opposite to the weight of the object (10×980 dynes).

$$\text{Hence work against gravity} = F \times s = 10 \times 980 \times 100$$
$$= 980,000 = 9.8 \times 10^5 \text{ ergs}$$

This work is exactly the same whether we raise the body vertically or pull it up a *smooth* inclined plane. (Does this apply to a plane of any inclination?)§

It is important to notice that the plane must be smooth. If this is not the case, not only should we have to do work against gravity but also against friction. Suppose the force of friction between the object and the plane is 1,100 dynes. Then the force parallel to the plane which you must use to move the object is 4,900+1,100, i.e., 6,000 dynes. The work done in pulling the object up the plane is

$$F \times s = 6,000 \times 200 = 1,200,000 = 12 \times 10^5 \text{ ergs}$$

(How much work is done against friction in pulling the object up the plane?) (*Answer:* 2.2×10^5 ergs.)

WORK AGAINST INERTIA

(WORK REQUIRED TO SET AN OBJECT IN MOTION)

LET US find how much work the husky does in giving the sled a velocity v. We shall suppose the ice is smooth, hence *no* work is done against friction. (While the fact that the ice is smooth does not in practice com-

pletely eliminate the force of friction, it reduces it so much that for the present purpose we shall take it as zero.) Also the ice is horizontal, so *no* work is done against gravity. You see why we prefer to conduct our experiments on ice.

Suppose the sled of mass m is originally at rest and that the dog pulls with a horizontal force F. The sled moves a distance s in a time t, acquiring a velocity v at the end of this time. Find the work required to give the sled this velocity v.

The work done by the horizontal force F in moving the sled a horizontal distance s is $F \times s$. But the force F gives the sled an acceleration a equal to F/m (from $F = ma$). The sled moving forward with this acceleration a covers a distance s in time t: $s = \frac{1}{2}at^2$.

$$\text{Work done} = Fs = ma \times s = ma \times \tfrac{1}{2}at^2 = \tfrac{1}{2}ma^2t^2$$

§ Answer to question on the previous page: No. It lightens the force of the sled on the ground.

The velocity v which the sled acquires after going for a time t with an acceleration a is equal to at.

Work done by husky in giving sled a velocity $v = Fs = \frac{1}{2}mv^2$

If the sled originally had a velocity v_i, in place of being at rest, and the husky pulled the sled over smooth ice a distance s, increasing the sled's velocity to v_f,† then the work done by the force F is

$$Fs = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

EXAMPLE. An automobile whose mass‡ is 900,000 gm. (2 ton) is traveling along a level road with a velocity of 440 cm. per sec. (10 MPH). The driver steps on the accelerator and

the car speeds up to a velocity of 2,640 cm. per sec. (60 MPH) in a distance of 9,000 cm. (100 yd.). Find the work done by the engine against the inertia of the car. Find the force exerted by the engine and also the acceleration of the car.

$$v_f = 2{,}640 \text{ cm. per sec.} \; ; \qquad v_i = 440 \text{ cm. per sec.}$$

$$\text{Work done by engine} = \tfrac{1}{2}m\,(v_f^2 - v_i^2) = \tfrac{1}{2} \times 900{,}000 \times [(2{,}640)^2 - (440)^2]$$
$$= 4.5 \times 10^5 [(2.64)^2 - (.440)^2] \times 10^6 = 4.5 \times 10^{11}(6.97 - 0.19)$$
$$= 4.5 \times 10^{11} \times 6.78$$
$$= 30.5 \times 10^{11} \text{ ergs}$$

This work done by engine = force exerted by engine × distance car moved

$$30.5 \times 10^{11} = F \times 9{,}000$$

$$\therefore \text{ Force } F = \frac{30.5 \times 10^{11}}{9{,}000} = 3.39 \times 10^8 \text{ dynes}$$

The force F exerted by the engine has given the car an acceleration a such that $F = ma$.

$$\therefore \quad a = \frac{F}{m} = \frac{3.39 \times 10^8}{900{,}000} = 3.77 \times 10^2 = 377 \text{ cm. per sec.}^2$$

§ Answer to question on the previous page: Yes.

† Suppose the sled originally had a velocity of v_i. Then the dog pulled with a force F over a distance s increasing the velocity of the sled to v_f.

$$\text{Work done} = Fs = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

In this case the sled has the velocity v_i when the force F is applied. This force F acting for a time t changes the velocity from v_i to v_f. During this time the sled moves a distance s. The work done by the force F moving a distance $s = Fs$. But $F = ma$, and acceleration $a = \dfrac{\text{change in velocity}}{\text{time}} = \dfrac{v_f - v_i}{t}$.

$$\text{Distance traveled: } s = \bar{v}t = \frac{v_i + v_f}{2} \times t$$

$$\therefore \text{ Work done: } Fs = mas = m \times \frac{v_f - v_i}{t} \times \frac{v_i + v_f}{2} \times t = \frac{m}{2} \times (v_f^2 - v_i^2)$$

$$\text{Work done in changing velocity of sled from } v_i \text{ to } v_f = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

If the initial velocity v_i is zero, this expression reduces to $Fs = \frac{1}{2}mv^2$.

‡ The British equivalents given in parentheses throughout the book are only approximately correct.

Thus the car, originally traveling with a velocity of 440 cm. per sec., acquires a velocity of 2,640 cm. per sec. due to the force of 3.39×10^8 dynes acting on it for a distance of 9,000 cm. The work done, 30.5×10^{11} ergs, has gone into speeding up the car, none of it being spent in overcoming friction or gravity.

ENERGY: KINETIC AND POTENTIAL

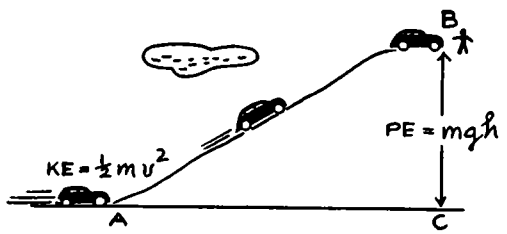
THE CAR just described acquired *energy*. The ENERGY of any object is defined as its CAPACITY FOR DOING WORK. Energy and work are measured in the same units, namely, ergs. The car with its velocity of 2,640 cm. per sec. can now climb a hill without any more force being applied by the engine, the work against gravity and friction being done at the expense of its energy. Work is done only when *energy is* EXPENDED. THE ENERGY WHICH AN OBJECT POSSESSES BY VIRTUE OF ITS VELOCITY IS CALLED ITS KINETIC ENERGY AND IS MEASURED BY $\frac{1}{2}mv^2$:

$$KE = \tfrac{1}{2}mv^2$$

Thus the result we proved on page 51, $Fs=\frac{1}{2}m\ (v_f^2-v_i^2)$, means that the work done against the inertia of an object is equal to the change in the kinetic energy of the object. The work done on the car is 30.5×10^{11} ergs and the kinetic energy has changed from 0.9×10^{11} ergs (i.e., $4.5\times10^{11}\times.19$ ergs) to 31.4×10^{11} ergs (i.e., $4.5\times10^{11}\times6.97$ ergs).

The car by virtue of its velocity of 2,640 cm. per sec. possesses 31.4×10^{11} ergs of kinetic energy. Let us find how big a hill the car can climb with this energy, assuming that we can neglect any frictional forces, the energy being entirely spent in overcoming gravity. We must put the car in neutral gear—we are not going to let the engine do any work in this experiment.

EXAMPLE. Let AB be the hill the car can just climb under these conditions. The velocity of the car at A is 2,640 cm. per sec. while its velocity at B is zero. In a preceding section we showed that if we could *neglect friction* the work required to take the car along the slope AB is exactly the same as that required to raise it vertically a distance BC. The work against gravity required to raise it a height BC is $F\times s$, i.e., $mg\times BC$, and this work is to use up all the kinetic energy which the car possessed at A. We have already found that



$$mg\times BC=31.4\times10^{11}\text{ ergs}$$

$$\therefore\ 900{,}000\times980\times BC=31.4\times10^{11}$$

$$BC=\frac{31.4\times10^{11}}{9.0\times10^5\times0.98\times10^3}=3.56\times10^3$$

$$=3{,}560\text{ cm.}$$

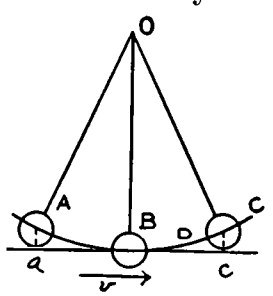
The car, at rest at B, has no velocity and hence no kinetic energy; still it has some sort of energy, for it can coast down the hill and gain in kinetic

energy exactly the amount of energy which it had when it was at rest at the top of the hill. THE ENERGY WHICH AN OBJECT POSSESSES BY VIRTUE OF ITS POSITION IS CALLED POTENTIAL ENERGY (PE). In the case of the gravitational force the potential energy of an object will be measured by the work against gravity which has to be done on the object to put it in its position. Thus if we raise an object of mass m gm. a height of h cm. we do mgh ergs of work against gravity and increase the PE of the object by this amount:

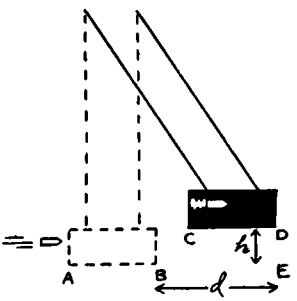
$$\text{Gravitational } PE = mgh \text{ ergs}$$

Although there is no absolute zero of PE, this will make no difference as we shall always be measuring the *difference in PE* brought about by the object being moved through some definite vertical height. Of course an object may possess PE due to other causes than the fact that it has been raised through some definite vertical distance. Thus, for example, the winding of your alarm clock at night gives the spring PE which is changed next morning to most objectionable sound waves. We shall encounter later the PE of an electric charge in an electric field.

THE TRANSFORMATION of PE into KE and vice versa is illustrated very well by a simple pendulum. A simple pendulum consists of a heavy bob of m gm. mass suspended by a light string as at OB. The pendulum bob released from A will swing backward and forward between A and C. At A and C the velocity and hence the KE of the bob is zero. As the bob falls from A to B it loses the PE which it has at A and this is transformed into KE. If we may neglect any frictional forces then, just as in the smooth inclined-plane example, the KE ($\frac{1}{2}mv^2$) which the bob has at B is all transformed into PE (mgh) at C so that $\frac{1}{2}mv^2 = mgh$, where $h = Cc$ or Aa. At some intermediate point such as D, the energy of the bob is partly kinetic and partly potential, but the sum of the two is equal to the PE at A (or C) or the KE at B. Thus the energy of the bob remains constant during these oscillations. So long as no energy is lost in work against friction, the total energy of the bob is conserved. That is, no energy is lost or gained. This is an example of the principle of CONSERVATION OF MECHANICAL ENERGY. This principle states that IN ANY MECHANICAL SYSTEM IN WHICH THERE ARE NO FRICTIONAL FORCES THE TOTAL ENERGY ($KE+PE$) IS CONSTANT. This principle follows from Newton's laws and was stated by Newton in 1687.



WE SHALL now take up the problem of determining the velocity of a bullet by shooting it into a block of wood suspended as a pendulum. This will involve the application of the Law of Conservation of Momentum and the principle of Conservation of Mechanical Energy. The block of wood is suspended by two strings so that when it swings it remains horizontal. The block is thick enough to stop the bullet. The bullet, whose mass is 5 gm., is fired into a block of wood whose mass is 6,995 gm. and which is suspended by two cords 245 cm. in length. The bullet penetrates the block of wood and causes it to swing from AB to CD, a horizontal distance of 10.0 cm. Find the velocity of the bullet.



The bullet makes an inelastic collision with the block of wood, as a result of which the bullet and block start to swing as a pendulum.|| Let V (cm. per sec.) be the velocity of the bullet just before it strikes the block and v (cm. per sec.) the velocity with which the bullet and block start to swing immediately after the collision.

The momentum of bullet and block before impact equals the momentum of bullet and block after impact (Law of Conservation of Momentum).

Before impact: Momentum of bullet $= 5V$, of block $= 0$

After impact: Momentum of bullet $= 5v$, of block $= 6{,}995v$

$$\therefore \quad 5V + 0 = 5v + 6{,}995v = 7{,}000v$$

$$\therefore \quad V = 1{,}400v$$

The velocity of the bullet before collision is fourteen hundred times as great as that of the bullet and block after the collision.

The bullet and block now start to swing as a pendulum. Owing to its KE, the block is raised a vertical height DE. From the principle of Conservation of Mechanical Energy, the KE of the bullet and block at AB is the same as its PE in position CD.

$$\therefore \quad \tfrac{1}{2}mv^2 = mgh; \qquad v^2 = 2gh$$

To find h when we know that the horizontal displacement d is 10 cm. and the

|| We have already considered a case of inelastic collision in the last chapter, so the first part of this problem is a review of the application of the Law of Conservation of Momentum.

length R of the supporting strings is 245 cm. requires the use of a little geometry.**

The relationship is $h=\frac{d^2}{2R}=\frac{10\times 10}{2\times 245}$, so that $v^2=2gh=\frac{2\times 980\times 10\times 10}{2\times 245}=$ 400 (cm. per sec.)^2. Therefore v, the velocity of bullet and block immediately after impact, is 20 cm. per sec. But we found from the Law of Conservation of Momentum that the velocity of the bullet is fourteen hundred times the velocity of bullet and block immediately after collision. Therefore velocity of bullet $V=1{,}400\times 20=28{,}000$ cm. per sec.

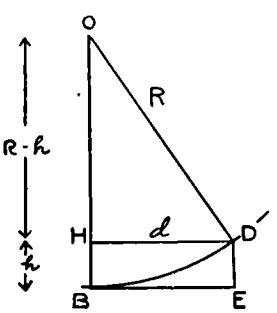
WE HAVE applied the principle of Conservation of Mechanical Energy to the bullet and block after the bullet has lodged in the block. Then why could we not apply this principle directly to the bullet and block before impact? As an exercise, calculate the energy of the bullet before impact. You should find it to be 1.96×10^9 ergs. The energy of the bullet and block after impact is 1.4×10^6 ergs, not at all the same as that of the bullet. In this case there is friction. Otherwise the bullet would not have come to rest in the block. Hence the principle of the Conservation of Mechanical Energy cannot be applied.

Has this energy $(1{,}960\times 10^6)-(1.4\times 10^6)$, i.e., 1958.6×10^6 ergs, been lost? The answer to this question was not given until about the middle of the nineteenth century, when it was proved experimentally by Joule that for every erg of work or energy lost by friction an equivalent quantity of heat appears. Probably if you cut the block of wood and dug out the bullet you would find it had been melted by the heat which appeared as the result of the loss of mechanical energy. Joule found from his experiments that to produce 1 calorie (cal.) of heat (to raise the temperature of 1 gm. of water 1° C.), 42,000,000 (4.2×10^7) ergs of work must be expended.

If we add the heat energy to the kinetic energy of the bullet and block,

** Let O be the center of a circle of radius R. Then $OB=R=OD$, and BD is an arc of the circle. OB is vertical. We wish to find what the vertical displacement, $DE=HB=h$, is in terms of the radius of the circle R and the horizontal displacement, $HD=d$. The length h is very small compared to R. By Pythagoras' theorem:

$$OD^2=OH^2+HD^2$$
$$R^2=(R-h)^2+d^2=R^2-2Rh+h^2+d^2$$
$$\therefore\quad 2Rh-h^2=d^2$$



But h is a very small quantity and in such a case the term h^2 is negligible compared to $2Rh$ or d^2. So we can take

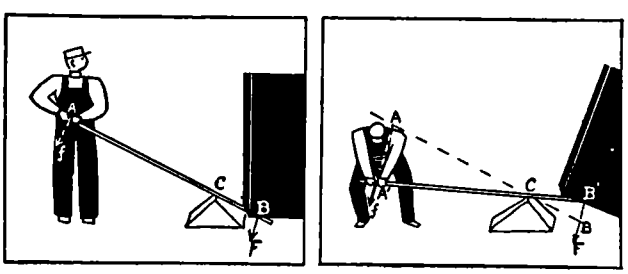
$$2Rh=d^2 \qquad \therefore\quad h=\frac{d^2}{2R}$$

we should find that together they equal the kinetic energy of the bullet before impact. Energy is again conserved. None is lost or gained at impact. This principle of the CONSERVATION OF ENERGY applicable to all energy transformations was first formulated by Mayer in 1845. It may be stated as "*Energy cannot be created or destroyed,*" or "*When energy in one form disappears, energy in some other form appears.*" This principle is of tremendous importance. It tells us that the total amount of energy in the universe is a constant. Stated in this manner, we must recognize energy in its variety of forms, light, sound, chemical, electrical, etc., all of which can be transformed into heat energy. You can appreciate now why in our statement of the principle of Conservation of *Mechanical* Energy we had to exclude frictional forces. *When work is done against friction, this work goes into heat energy.*

The fact that energy cannot be created by any machine which man's brain can think up tells us immediately that so-called "perpetual motion" machines can never be made to supply energy from nothing. It is interesting to note in this connection that the French Academy of Science, in 1775, decided not to waste any more of its time on inventors of "perpetual motion" machines. However, such machines are still being made by people whose self-confidence is greater than their knowledge of physics.

BY THIS time you are probably willing to agree that there is another form of energy, intellectual energy, and we suggest that rather than waste your time attempting to make a "perpetual motion" machine you measure the number of calories of heat produced by your working of one of the problems. We, ourselves, shall take the much simpler problem of finding the number of calories produced when work is done against frictional forces, as in Joule's experiments. However, we must lead up to this with another problem.

YOU HAVE undoubtedly heard about levers and maybe know already the principle on which they work. In the diagram the man is attempting to use a force f to overcome a much larger force F. He uses a lever AB which can turn about a point C, this point C being called the *fulcrum.* In moving the lever from the position AB to the position $A'B'$, the man does an amount of work $F \times s$, or $f \times AA'$, while an amount of work is done against the *load* of $F \times s$, or $F \times BB'$. If there are no frictional



forces and we neglect the weight of the lever, the amount of work done by the man, $f \times AA'$, must be the same as the amount of work done against the load, $F \times BB'$.

$$f \times AA' = F \times BB'$$

The reason why the man uses the lever is that he is unable to raise the block of stone alone. The lever has given him an advantage: He is now able to overcome a large force F by using a small force f. This advantage is equal to the ratio F/f. It is called the MECHANICAL ADVANTAGE. The force overcome, F, is often called the LOAD, while the force used, f, is called the EFFORT.

$$\text{Mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

However, what is gained in force is lost in distance, for the load is moved through a proportionally smaller distance than is the effort.

IN THE diagram of the lever the triangles ACA' and BCB' have corresponding angles equal, i.e., the triangles are similar. We can apply a theorem from geometry about similar triangles. The ratio of corresponding sides of the one is equal to the ratio of corresponding sides of the other:

$$\frac{AA'}{A'C} = \frac{BB'}{B'C} \quad \text{or} \quad \frac{AA'}{BB'} = \frac{A'C}{B'C}$$

The relationship $f \times AA' = F \times BB'$ may be written $\frac{AA'}{BB'} = \frac{F}{f}$, so that

$$\frac{A'C}{B'C} = \frac{F}{f} \quad \text{or} \quad f \times A'C = F \times B'C$$

The distance from where the effort is applied to the fulcrum is the *lever arm* of the effort f. This lever arm is $A'C$ in the diagram, while the lever arm of the load F is $B'C$. The product $f \times A'C$ measures the tendency of the effort f to turn the lever about the fulcrum C and is called the *moment of* f about C. THE MOMENT OF A FORCE ABOUT A POINT IS THE PRODUCT OF THE FORCE AND THE PERPENDICULAR DISTANCE FROM THE POINT TO THE LINE OF THE FORCE. Thus the result $f \times A'C = F \times B'C$ means that the moment of the effort f about C is equal to the moment of the load F about C.* An ordinary chemical balance illustrates this principle, but usually in this case the lever arms

* In the case we have chosen, the effort f caused the lever to rotate. This must mean that the moment $f \times A'C$ is greater than the moment $F \times B'C$. This is true when the lever starts to move, but once motion is started, $f \times A'C$ must equal $F \times B'C$ because there is to be no angular acceleration.

are made very accurately equal. Does a balance compare masses or weights?§ Consider what would happen if the acceleration of gravity were suddenly doubled.

EXAMPLE 1. Let us take a simple problem involving the use of moments. A meter stick can turn freely about the fulcrum C which is at a point 80 cm. from one end and 20 cm. from the other. A 50 gm. wt. hangs on the end 80 cm. from the fulcrum. What weight, W, must be hung at the other end to balance this weight? We shall neglect the weight of the meter stick.

Moment of 50 gm. wt. about $C = 50 \times 80 = 4{,}000$ gm. wt.$\times$cm.

Moment of W gm. wt. about $C = W \times 20$ gm. wt.$\times$cm.

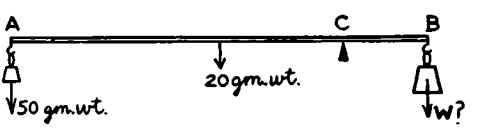
When the stick balances these moments are equal:

$$W \times 20 = 4{,}000 \qquad \therefore \quad W = 200 \text{ gm. wt.}$$

Suppose we cannot neglect the weight of the meter stick. If we take the meter stick alone we shall find that it balances about its midpoint. This point, where the whole weight of the stick appears to act, is called its *center of gravity, CG*. The *CG* of a homogeneous symmetrical object is at its geometrical center.

EXAMPLE 2. Find the weight which must be hung on the end B if the meter stick weighs 20 gm.

The weight of the meter stick acts 50 cm. from each end. This will be 30 cm. from the fulcrum.



Moment of the 50 gm. wt. about $C = 50 \times 80 = 4{,}000$ gm. wt.$\times$cm.

Moment of the 20 gm. wt. about $C = 20 \times 30 = 600$ gm. wt.$\times$cm.

Both these moments tend to make the stick move counter-clockwise about C:

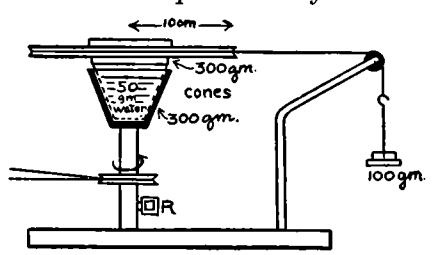
Total counter-clockwise moment about $C = 4{,}600$ gm. wt.$\times$cm.

This must be balanced by the clockwise moment of the weight W about C, which is equal to $W \times 20$.

$$\therefore \quad W \times 20 = 4{,}600; \qquad W = 230 \text{ gm. wt.}$$

It is reasonable to expect that the weight required here (230 gm. wt.) will be larger than in the case where the stick was assumed to have no weight.

WE SHALL now return to the problem of experimentally determining the *mechanical equivalent of heat J* by a modification of Joule's apparatus.



Two hollow cones, so-called friction cones, fit snugly one into the other. The lower one is made to rotate while the upper one is held still. The two conical surfaces rub against each other and heat is produced. This heat warms up the friction cones and the water which has been placed in the upper cone; the increase of temperature is measured by a thermometer placed in the water.

The upper cone is rigidly connected to a horizontal wooden disk which is prevented from turning by a weight attached to the disk. This string passes over a pulley to the hanging weight. The speed of turning must be adjusted so that the hanging weight remains at rest. If the force of friction between the cones is too large for the weight, the outer cone will tend to rotate and the hanging weight will wind up. On the other hand, if the friction is too small, the hanging weight will fall down. When the hanging weight remains suspended at rest, the tendency of the frictional force to wind up the hanging weight is balanced by the tendency of the weight to fall. Now the moment of the force tending to make the weight fall down is the product of the weight and the radius of the disk. For one complete revolution of the lower cone, the work done against friction will be measured by the product of the weight and the circumference of the disk. We cannot measure the frictional force between the cones, for it is acting along the whole surface of contact between the cones, but we can measure the work done by it in one revolution by using the disk and the hanging weight.

EXAMPLE. Suppose in an experiment of this kind each of the brass cones has a mass of 300 gm. and that the upper one contains 50 gm. of water. The disk has a radius of 10 cm. and is prevented from rotating by a hanging weight of 100 gm. After rotating the cone one thousand times (as measured by a revolution counter R), the water has increased in temperature 1.4° C.

For 1 revolution, the work done against friction as measured by the disk and hanging weight:

$$W = 100 \times 2\pi \times 10 \text{ gm. wt. cm.} = 100 \times 2\pi \times 10 \times 980 \text{ ergs}$$

For 1,000 revolutions work done:

$$W = 1{,}000 \times 100 \times 2\pi \times 10 \times 980 = 1{,}000 \times 100 \times 2 \times 3.14 \times 10 \times 980$$

$$(\pi = 3.14) \qquad = 6.15 \times 10^9 \text{ ergs}$$

This energy has gone into raising the temperature of the two cones and the water 1.4° C. It requires 1 cal. of heat to raise the temperature of 1 gm. of water 1° C., so that it will require 50×1.4, or 70 cal., to raise the temperature of 50 gm. of water 1.4° C. To raise the temperature of 1 gm. of brass 1° C., 0.09 cal. of heat are required, so that to raise the temperature of 600 gm. of brass 1.4° C. will require $600 \times 0.09 \times 1.4$, or 74.6 cal. The total number of calories produced is $70 + 74.6$, or 144.6 cal.

$$\text{Mechanical equivalent of heat } J = \frac{\text{work against friction}}{\text{heat produced}} = \frac{6.15 \times 10^9}{144.6}$$

$$= 4.2 \times 10^7 \text{ ergs per cal.} = 4.2 \text{ joules per cal.}$$

(The accepted value for J is 4.187×10^7 ergs per cal.)

PROBLEMS

1. If you carry a bag whose mass is 50,000 gm. along a level road for a distance of 100 m, how much work do you do against gravity?

§ Answer to question on the previous page: Masses.

2. A boy throws a ball whose mass is 100 gm. 20 m. vertically into the air.
   a) What is the *PE* of the ball at the top of its flight?
   b) What is the *KE* of the ball when it leaves the boy's hand?
   c) What is the work done by the boy in throwing the ball?

3. A man whose mass is 70 kg. walks up to the third floor, which is a vertical height of 12 m. above street level.
   a) How many ergs of work has he done?
   b) How much has he increased his *PE?*
   c) If he climbs the stairs in 20 sec., how may watts has he expended?

4. Against what does a car do work when it travels along a level road with uniform velocity?

5. Against what does a car do work when it speeds up on a level road?

6. Against what does a car do work when it accelerates up a hill?

7. A sled whose mass is 15,000 gm. is pulled with uniform velocity over a rough but level spot where the force of friction on the sled is 1,000 gm. wt. How many ergs of work are done when the sled is moved 10 m.?

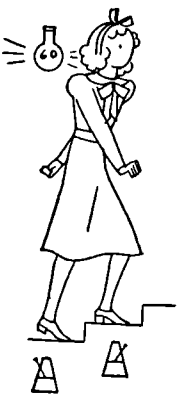
8. A boy pulls a sled by means of a rope which makes an angle of 30° with the sled. If the boy pulls on the rope with a force of 1,000 gm. wt., how much work does he do when the sled is moved over level ground a distance of 20 m. with a constant velocity of 30 cm. per sec.?

9. A rifle whose mass is 2,500 gm. fires a bullet whose mass is 5 gm. with a velocity of 20,000 cm. per sec. Find
   a) The *KE* of the bullet.
   b) The work done by the powder in sending the bullet out.
   c) The velocity of recoil of the rifle.
   d) The *KE* of the rifle.
   The bullet enters a block of wood 2 cm. thick and emerges on the other side with a velocity of 10,000 cm. per sec. Find
   e) The amount of *KE* lost by the bullet in going through the wood.
   f) The work done by the bullet against friction while going through the wood.
   g) The frictional force exerted by the wood.

10. A boy whose mass is 50 kg. takes a running start and slides on a slippery sidewalk. He starts sliding with a velocity of 4 m. per sec. and slides a distance of 5 m. before coming to rest. Find
    a) The *KE* of the boy when he starts sliding.

b) The work done by the frictional forces in bringing him to rest.
c) The value of the frictional force which brings him to rest.

11. A girl whose mass is 40 kg. runs up 100 steps each 20 cm. high in 50 sec. Find
a) The work done against gravity by the girl.
b) The increase in *PE* of the girl.
c) The power in watts expended by the girl.



12. In loading a freight car from a railway platform, boxes having a mass of 10^6 gm. are slid up an inclined plane whose length is 10 m. and whose vertical height is 2 m. Find
a) The increase in *PE* of the boxes in the freight car after being lifted from the railway platform.
b) The work done in pulling the boxes up the inclined plane if there is no friction.
c) The force necessary to pull the box up the plane if there is no friction.
If the force of friction between a box and the plane is 0.54×10^8 dynes, find
d) The work done in pulling a box into the freight car.
If one of the boxes falls off the freight car onto the platform, find
e) Its *KE* when it hits the platform.

13. A boy throws a baseball whose mass is 150 gm. a vertical height of 15 m.
a) How much work does the boy do in throwing the ball?
b) What is the *KE* of the ball when it returns and hits the ground?

14. A hammer whose mass is 2,500 gm. has a velocity of 1,000 cm. per sec. when it strikes a nail which it drives 0.5 cm. into a block of wood. Find
a) The *KE* of the hammer just before it strikes the nail.
If all this energy is spent in driving the nail into the wood, find
b) The resisting force of the wood.
c) The calories of heat produced.

15. A child's wagon having a mass of 10,000 gm. is pulled along with a horizontal force giving it an acceleration of 30 cm. per sec.^{2}. If the resistance forces amount to 700,000 dynes, find
a) The total horizontal force which must be used.
b) The total work done when the wagon has been moved 15 m.
c) The work done against the frictional forces.

d) The heat in calories produced by friction.
e) The *KE* of the wagon after it has been pulled 15 m.
f) The velocity of the wagon after it has been pulled these 15 m.

16. An automobile having a mass of 5×10^5 gm. climbs, with uniform velocity, a hill 1,000 m. long and 50 m. high. Find
a) The *PE* of the car at the top of the hill.
If the resisting forces due to friction amount to 1.65×10^7 dynes, find
b) The total work required to get the car to the top of the hill.

17. Two steel balls, each having a mass of 50 gm., are suspended by long cords of equal length so that when they are at rest the two balls just touch. One of the balls is pulled aside and then let go. It strikes the stationary ball with a velocity of 30 cm. per sec. If the collision is perfectly elastic, find
a) The velocity of each of the balls immediately after impact.
b) The energy of the balls before and after the collision.
If the collision is made inelastic by placing a piece of plasticene between the balls, find
c) The energy before and after impact.

18. A boy catches a ball whose mass is 150 gm. which is traveling with a velocity of 200 cm. per sec. In doing so he draws his hands back 10 cm.
a) How long is the ball in the boy's hands before it is brought to rest?
b) What force does the boy exert in bringing the ball to rest?
c) What is the *KE* of the ball just before the ball enters the boy's hands?
d) How much work does the boy do in bringing the ball to rest?
e) If the boy had only drawn his hands back 5 cm., would he do the same amount of work in bringing the ball to rest as he did when he drew his hands back 10 cm.?
f) Find the average force exerted by the boy when he draws his hands back 5 cm.

19. *a*) The engines of an airplane develop 1,000 h-p. How many watts do they develop?
b) If the airplane has a mass of 2×10^6 gm., find how high it can rise each second, assuming that all the energy goes into climbing.

20. When rowing a boat you use an oar 2 m. long. The distance from the handle of the oar to the oarlocks is 50 cm., so the distance from the blade of the oar to the oarlocks is 150 cm.
a) If the oar is regarded as a lever, where is its fulcrum?

b) Where are the effort and the load applied?

c) If you pull with a force of 2.5×10^4 gm. wt., find the force with which the boat is urged forward.

d) What is the mechanical advantage of this oar?

21. In cracking a nut with a pair of nutcrackers it was necessary to squeeze with a force of 10^4 gm. wt. If the distance from a handle of the nutcracker to the nut is 10 cm. and the distance from the nut to the hinged end of the nutcracker is 1 cm.,

a) What is the force required to break the nut?

b) What is the mechanical advantage of the nutcracker?

22. You try your strength at a fair by striking a wooden block with a hammer. This shoots a weight up a vertical slide. If the weight goes high enough, a bell rings and you get a cigar.

a) If the sliding weight has a mass of 2×10^3 gm. and the bell is 15 m. high, how much energy must the weight have to just reach the bell?

b) If the hammer has a mass of 10^4 gm. and if all of its energy is transferred to the sliding weight (when it strikes the wooden block), what velocity must the hammer have to make the weight strike the bell?

23. Water falling from a height of 300 cm. at a rate of 2.5×10^5 gm. per sec. is used to turn a water wheel.

a) If all the energy of the water goes into turning the wheel, what power, in watts and in horse-power, is developed?

b) If, as would be more probable, only 0.25 of the energy of the water goes into turning the wheel, what horse-power is developed.

24. Water naturally flows downhill, so that the farther downstream you go the lower the water-level.

a) Would you expect a river to flow faster the farther you go downstream?

b) If this is not actually the case, can you say what has become of some of the *PE* of the water?

25. A man at rest in a rowboat starts to pull himself to shore by pulling on a long rope tied to a pier. The man and the boat have a total mass of 2×10^5 gm. The man pulls on the rope with a constant force of 10^6 dynes.

a) What velocity would he attain in 3 sec. if there were no friction?

b) How far does he move in these 3 sec. and how much work has he done?

c) What is the *KE* of the boat and man at the end of 3 sec.?

d) The rope, instead of being tied to the pier, is tied to another boat whose mass is 4×10^5 gm. Find the velocity of each of the boats at the end of 3 sec.

e) Find the work done by the man and also the total *KE* of each of the two boats. Again neglect friction. (The man keeps a constant tension in the rope.)

f) Does he pull more rope through his hands in the second case than in the case when the rope is tied to the pier?

g) Does this account for the fact that the man does more work in the second case than in the first?

26. A rifle whose mass is 2,500 gm. fires a bullet into a thick block of wood which is suspended by two long strings each having a length of 980 cm. The block of wood has a mass of 9,995 gm. The bullet, which has a mass of 5 gm., comes to rest in the wood, causing it to swing a horizontal distance of 3 cm.

a) Find the velocity of the bullet.

b) Find the *KE* of the bullet as it strikes the wood.

c) Find the energy of the bullet and block when they start to swing.

d) How many calories of heat are produced?

27. By measuring the difference in temperature of the water at the top and bottom of a waterfall, it is possible to get a value for *J*, the mechanical equivalent of heat. For Niagara Falls, with a drop of about 50 m., this difference in temperature should be found to be about 0.11° C. Consider a 1 gm. mass of water falling over the waterfall.

a) How much *PE* does this 1 gm. of water lose in falling to the bottom?

b) How much *KE* does this 1 gm. of water gain in falling to the bottom?

c) How many calories of heat has this 1 gm. of water gained in falling the 50 m.?

d) What, then, is the mechanical equivalent of heat?

e) Should the temperature of the water at the bottom of the falls be measured just before it reaches the bottom, in the eddies produced directly at the bottom, or in the steady stream which flows along after the eddies have been left behind?

28. A rough determination of the mechanical equivalent of heat may be made by using a long cardboard tube closed at both ends and partly filled with lead shot. The tube is quickly turned upside down a number of times, allowing the lead shot to fall to the bottom each time. The temperature of the shot is taken before and after the experiment.

Suppose such a tube is 1 m. long and contains 20 gm. of lead shot. The tube is turned upside down fifty times. If 2.0 cal. of heat are produced as measured by the increase in temperature of the lead shot, find

a) The *PE* of the shot when at the top of the tube.

b) The *KE* of the shot when it falls to the bottom of the tube.

c) The mechanical equivalent of heat.

If the accepted value for the mechanical equivalent of heat is taken as 4.2×10^7 ergs per cal.,

d) Can you say where some of the heat, i.e., that not used to bring about the rise in temperature of the lead shot, has gone?

29. You are in a roller-coaster car and are towed up an incline a height of 30 m. The car and contents have a total mass of 5×10^5 gm. The car coasts to ground level.

a) What is the *KE* of the car when it reaches the bottom, assuming that friction may be neglected?

b) How high an incline can the car climb with this energy, again assuming no friction?

c) If we assume that we may neglect friction on the downgrade, but that on the upgrade 14.7×10^{10} ergs of energy is spent in overcoming friction, find how high a slope you can climb.

30. An automobile whose mass is 10^6 gm. (1 ton) is at rest on a level road. The driver steps on the accelerator and the car speeds up to a velocity of 2,500 cm. per sec. in a distance of 10,000 cm. Find

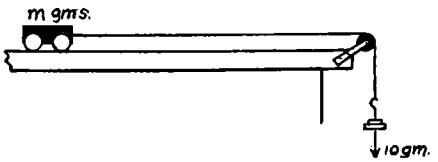
a) The work done by the engine against the inertia of the car.

b) The force exerted by the engine and the acceleration of the car.

c) How high a hill the car can climb in neutral gear with the energy which it possesses (neglect friction).

31. A hundred bullets a second are shot with a velocity of 25,000 cm. per sec. at a steel plate. Each bullet has a mass of 10 gm. Assuming that the collisions between the bullets and the plate are elastic, find the force needed to hold the plate at rest.

32. A 10 gm. wt. hung vertically is attached by a string to a frictionless car which runs along a level track. The car, whose mass is m gm., covers a distance of 125 cm. in 5 sec. after it starts from rest. Find



a) The average and final velocity of the car.

b) The loss in PE of the 10 gm. wt. in falling through 125 cm.

c) The gain in KE of the car (mass m) and the hanging weight.

d) The mass of the car.

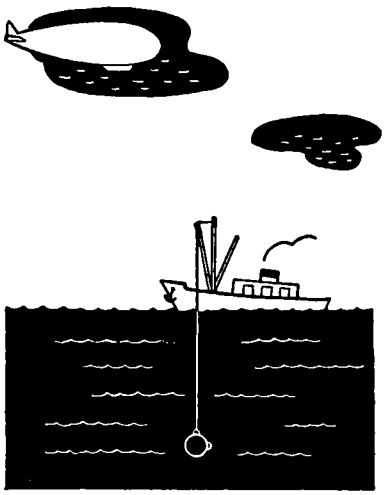
33. Why is it easier to close a door by pushing near the knob than by pushing near the hinges?

34. An automobile crashes into a telephone post and is brought to rest.

a) Where does the momentum of the automobile go?

b) Does the earth receive any KE as a result of the crash?

c) Where does by far the greater part of the KE of the automobile go?



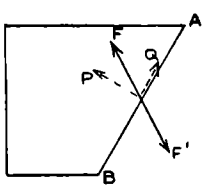
CHAPTER 6

ATMOSPHERE AND HYDROSPHERE

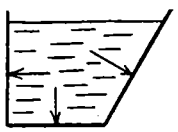
ONE THING which is surprising when you see Beebe's bathysphere is the great thickness of the steel shell. This is necessary in order to withstand the enormous pressure of the water at the depths of the ocean to which it descends. The water presses on the sides of the bathysphere, the pressure increasing with the depth. The pressure at any point in the ocean is due to the weight of the water above the point. We speak of it as a GRAVITY PRESSURE.

IF YOU were asked for a difference between *fluids* (i.e., liquids and gases) and *solids* you might say that in order to hold a fluid some sort of a vessel is necessary. Fluids have no rigidity. They always take the shape of the containing vessel. A FLUID IS A SUBSTANCE WHICH IS UNABLE TO OFFER ANY PERMANENT RESISTANCE TO FORCES TENDING TO CHANGE ITS SHAPE.

A fluid at rest exerts a force on any surface in contact with it. This force must always be at right angles to the surface. In order to show this we shall consider the liquid in the vessel shown in the diagram. The liquid exerts a force on all sides of the vessel. If any one of the sides were removed the liquid would flow out. We shall make a supposition that the force F', which the liquid exerts on the side AB, is not at right angles to AB, and then see whether this can be true. By Newton's Third Law the force F which the surface AB exerts on the liquid must be equal and opposite to F': $F = -F'$. The force F can be resolved into two components, Q and P,

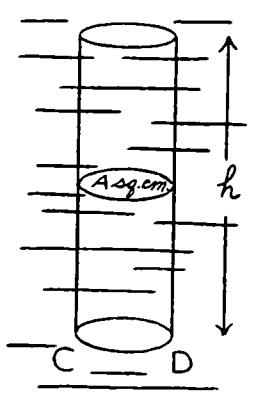


one along AB and the other at right angles to AB. The component Q would make the liquid *move* in the direction AB, but we have assumed that the liquid is *at rest*. Therefore there can be NO component Q along the surface AB. Hence the only force the surface can exert on the liquid must be at right angles to the surface, for this is the only force which has no component along the surface.



The amount of the force which a liquid exerts on the surface of an object immersed in the liquid depends on the area of this surface and the depth to which it is immersed. The deeper you go in a liquid, the greater the gravity pressure. BY PRESSURE WE MEAN THE FORCE ON UNIT AREA, and this will be expressed in dynes per sq.cm.: $p = F/A$, where F is the force exerted by the liquid on a surface of area A.

HOW DOES the gravity pressure in a liquid increase with depth? Imagine in the liquid a right cylinder of A sq.cm. in cross-section and h cm. in height. The volume of this cylinder is equal to the area of its base multiplied by its height, $V = hA$, and is expressed in cubic centimeters (cc.). To find the mass of liquid within the imaginary cylinder we must know the mass of 1 cc. of the liquid, i.e., its DENSITY. $Density = \frac{mass}{volume}$ and is expressed in gm. per cc. We shall let the density of this liquid be ρ gm. per cc. Thus the mass of the volume hA cc. is $hA\rho$ gm. and its weight is $hA\rho g$ dynes. This weight, $hA\rho g$ dynes, all acts on the base of the cylinder CD. But the liquid at the base does not move. Hence this liquid must be exerting an upward force of $hA\rho g$ dynes. But the pressure p on the base of the cylinder is the force exerted on the base, $hA\rho g$ dynes, divided by the area A of the base:



$$p = \frac{\text{force}}{\text{area}} = \frac{hA\rho g}{A} = h\rho g \text{ dynes per sq.cm.}$$

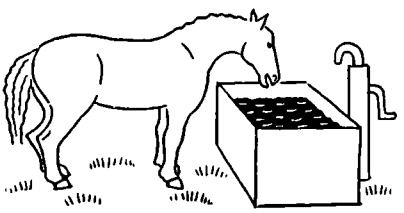
Thus the gravity pressure p at a depth h cm. in a liquid of density ρ gm. per cc. is $h\rho g$ dynes per sq.cm.

The atmosphere, as you must know, exerts a gravity pressure, due to the weight of air above. This pressure is less when you go to the top of a mountain because as you climb the weight of air above becomes smaller. Atmospheric pressure at sea-level is not quite constant but has an average value of about 1,000,000 (10^6) dynes per sq.cm.

EXAMPLE 1. Suppose you dive to a depth of 20 m. in water. What is the pressure which is exerted on you at this depth? Pure water has a density of 1 gm. per cc. The pressure at a depth of 20 m. of water due to the weight of the water above is

$$p = h\rho g = 20 \times 100 \times 1 \times 980 = 1.96 \times 10^6 \text{ dynes per sq. cm.}$$

This is almost twice atmospheric pressure. The total pressure exerted on you at a depth of 20 m. of water is almost 3 atmospheres (1 due to the atmosphere and 2 due to the water).



EXAMPLE 2. A rectangular drinking trough for horses is 120 cm. deep, 1.5 m. broad, and 3 m. long. Find the force due to the water which is exerted on the sides and on the bottom when the tank is filled to a height of 100 cm. with pure water.

Pressure on the bottom of the tank $= h\rho g$

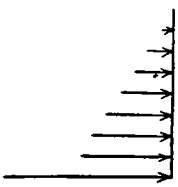
$= 100 \times 980 \times 1$ dynes per sq. cm.

Total force on bottom of tank $= pA$ = area of bottom $\times$ pressure

$= 300 \times 150 \times 100 \times 980 \times 1$

$= 3 \times 1.5 \times 0.98 \times 10^9$

$= 4.41 \times 10^9$ dynes



The pressure due to the water at the top of the water is 0, at the bottom is $100 \times 980 \times 1$ dynes per sq.cm. The pressure increases uniformly with depth, so we may say that, on the average, any side is subjected to a pressure:

$$\bar{p} = \frac{0 + 98{,}000}{2} = 4.9 \times 10^4 \text{ dynes per sq.cm.}$$

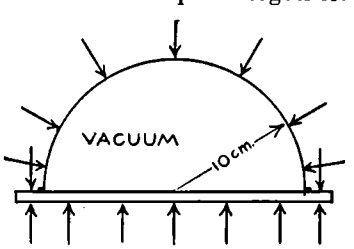
But the area of side $= 300 \times 100 = 3 \times 10^4$ sq.cm.

Force on side: $\bar{p}A = 4.9 \times 10^4 \times 3 \times 10^4 = 1.47 \times 10^9$ dynes

Area of end $= 150 \times 100 = 1.5 \times 10^4$ sq.cm.

Force on end $= \bar{p}A = 4.9 \times 10^4 \times 1.5 \times 10^4 = 7.35 \times 10^8$ dynes

EXAMPLE 3. A hemispherical shell of 10-cm. radius is placed on a flat plate and the space inside evacuated. Calculate the force holding the hemispherical shell and plate together. The pressure due to the atmosphere is everywhere at right angles to the shell, and it would be difficult to resolve all these forces and then multiply their vertical components by the corresponding areas. This is not necessary since the downward vertical force with which the atmosphere pushes the shell on the plate must be the same as the upward vertical force with which the atmosphere pushes the plate onto the shell. Why?§



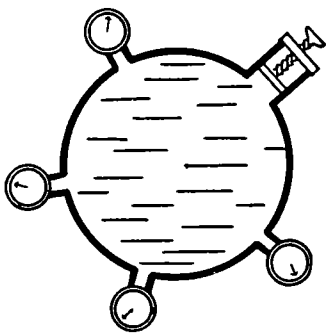
In the case of the flat plate the pressure is everywhere in the same direction. This pressure is atmospheric pressure, 10^6 dynes per sq.cm.:

Force pushing plate onto shell $= pA$; $A = \pi r^2 (\pi = 3.14)$

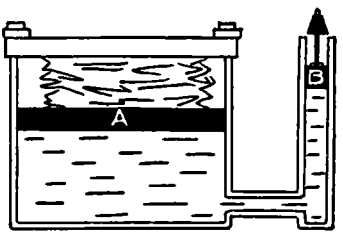
$pA = 10^6 \times 3.14 \times 10^2 \qquad = 3.14 \times 10^8$ dynes

A force of 3.14×10^8 dynes, or about 300 kg. wt., would be required to separate the shell from the plate.

TRANSMISSION OF PRESSURE: Suppose we have a liquid filling a strong closed vessel and apply pressure to the liquid by means of a piston or screw. Will this pressure be transmitted equally to all parts of the liquid? This question can be answered by the apparatus shown in the diagram. It will be found that when the pressure is changed all the pressure gauges record the same change. In other words, an increase of pressure in a liquid is equally transmitted to all parts of the liquid. This principle is applied in the hydraulic brakes of automobiles.



Another application of this principle of transmission of pressure in liquids is found in the so-called hydraulic press. A type of hydraulic press is shown in the diagram. A and B are watertight pistons. A weight placed on piston B increases the pressure in the liquid. This pressure is transmitted to the piston A. Thus A is subjected to a large force which compresses the material above A.



EXAMPLE. Let the area of piston B be 30 sq.cm. and the area of piston A be 300 sq.cm. If a weight of 10 kg. (980×10^4 dynes) be placed on B, what is the force on A?

The liquid is subjected to an increase of pressure due to the weight on B.

$$p=\frac{\text{force}}{\text{area}}=\frac{980\times10^4}{30}\text{ dynes per sq.cm.}$$

This pressure is transmitted through the liquid. Therefore

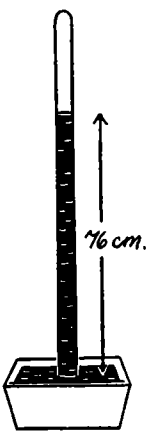
$$\text{Force on piston } A=\text{pressure}\times\text{area}=\frac{980\times10^4\times300}{30}$$
$$=980\times10^5\text{ dynes}$$

Thus the force on piston A is ten times the force on B. This is the ratio of the area of A to the area of B.

NOW WHEN a liquid is subjected to a large pressure the volume of the liquid changes by only a negligible amount; a liquid is almost incompressible. A gas is quite different: comparatively small changes in pressure change the volume considerably.

§ Answer to question on the previous page: Newton's Third Law.

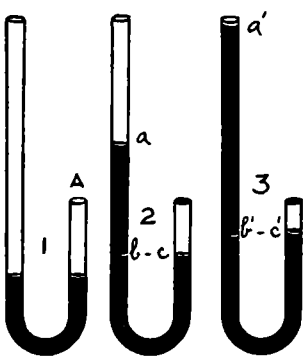
BEFORE DISCUSSING this change of volume with pressure let us see how atmospheric pressure can be measured. The most accurate method uses a *mercury barometer*. The lower open end of a glass tube about 1 m. long is placed under mercury contained in a dish and the upper end connected to a vacuum pump. Mercury will rise in the tube to a height of about 76 cm. If the upper end is then sealed off so that there is no air in it, we have a good barometer. The atmosphere, extending upward many miles, is now holding up this column of mercury 76 cm. high. The pressure due to the atmosphere is equal to the pressure at the bottom of a column of mercury 76 cm. high. Mercury Hg is a very dense liquid. One cubic centimeter of it has a mass of 13.6 gm. (ρ for $Hg = 13.6$ gm. per cc.). The pressure at the bottom of a column of mercury 76 cm. high is $hg\rho$:



$$hg\rho = 76 \times 980 \times 13.6 = 1.01 \times 10^6 \text{ dynes per sq. cm.}$$

This is the pressure of the atmosphere in dynes per sq.cm. For convenience atmospheric pressure is often expressed as the column of mercury it will balance. Thus in the present case we may say it is 76 cm. of mercury (76 cm. of Hg), meaning by this that the pressure is $76 \times 980 \times 13.6$ dynes per sq.cm. If for any reason the atmospheric pressure changes, the height of the column of mercury which it balances will change.

BOYLE'S LAW: THE RELATION BETWEEN PRESSURE AND VOLUME OF A GAS WHEN THE TEMPERATURE IS CONSTANT. In order to investigate this relationship let us use a U-tube with unequal arms, the shorter arm of which is sealed off. This shorter arm contains air and has for convenience a uniform cross-section. The air in the sealed arm of the U-tube is compressed to varying degrees by filling the long arm to different levels with mercury.



1. Let us start the experiment with the level of mercury in the two columns the same.

Pressure of air in short tube $= p =$ atmospheric pressure $= 76$ cm. of mercury
Volume occupied by air in short tube $= 12$ cc.

2. More mercury is poured into the long arm. This compresses the air in the short tube. The pressure on the air in the short tube is the same as the

pressure at the point b in the long tube, where c and b are in a horizontal line. The height of the mercury column ab is 38 cm., so that

$$\left.\begin{matrix}\text{Pressure at } b \\ \text{in cm. } Hg\end{matrix}\right\} = \left\{\begin{matrix}\text{atmospheric pressure} \\ \text{in cm. } Hg\end{matrix}\right\} + \left\{\begin{matrix}\text{height of column } ab \\ \text{in cm. } Hg\end{matrix}\right\}$$

$$= 76 + 38 = 114 \text{ cm. } Hg$$

Volume of air in short tube = 8 cc.

3. Still more mercury is poured into the long arm until the height $a'b'$ is 76 cm.

$$\left.\begin{matrix}\text{Pressure at } b' \\ \text{in cm. } Hg\end{matrix}\right\} = \left\{\begin{matrix}\text{atmospheric pressure} \\ \text{in cm. } Hg\end{matrix}\right\} + \left\{\begin{matrix}\text{height of column } a'b' \\ \text{in cm. } Hg\end{matrix}\right\}$$

$$= 76 + 76 = 152 \text{ cm.}$$

Volume of air in short tube = 6 cc.

We shall collect these results into a table from which we see clearly that as the pressure is increased the volume is decreased in the same ratio. Let

Pressure p in Cm. of Mercury	Volume V of Air in Cc.	pV
76	12	912
114	8	912
152	6	912

us now multiply the pressure by the volume. The product pV is 912 in each case. Thus the product of the pressure and volume of the gas is a constant, which we shall call K. This is true only when the temperature is constant. The result is known as *Boyle's Law*: THE PRESSURE OF A GIVEN MASS OF GAS KEPT AT CONSTANT TEMPERATURE IS INVERSELY PROPORTIONAL TO THE VOLUME.

$$pV = K; \qquad p = \frac{K}{V}$$

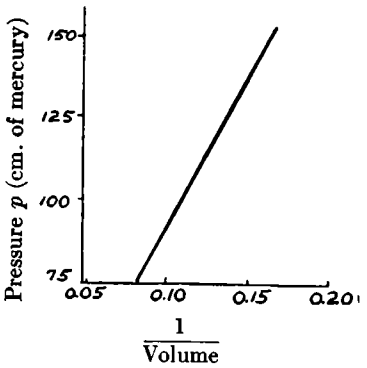
In the experiment described the constant K for the particular mass of gas kept at some constant temperature was 912. The result $pV = K$ may also be written

$$\frac{p_1}{p_2} = \frac{v_2}{v_1}.$$

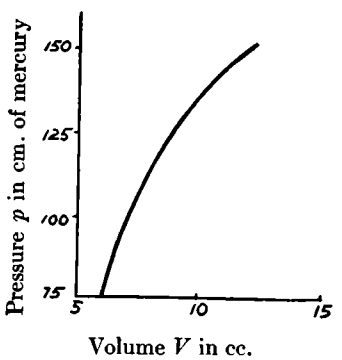
Thus taking our data, $p_1 = 76$ cm. of mercury, $v_1 = 12$ cc.; $p_2 = 114$ cm. of mercury, $v_2 = 8$ cc.:

$$\frac{p_1}{p_2} = \frac{76}{114} = \frac{2}{3} \qquad \frac{v_2}{v_1} = \frac{8}{12} = \frac{2}{3}$$

Since $p = (1/V) \times K$, we should get a straight-line graph if we plot p against $1/V$. If we plot pressure p against volume V, we get a curve quite different from a straight line. We can further analyze this constant K. Consider what



Pressure p in Cm. of Hg	Volume V in Cc.	$\frac{1}{\text{Volume}}$
76	12	$\frac{1}{12} = 0.083$
114	8	$\frac{1}{8} = 0.125$
152	6	$\frac{1}{6} = 0.167$



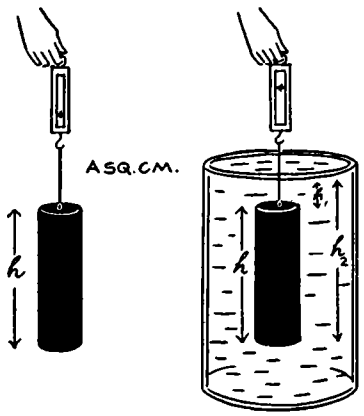
happens if we double the amount or mass of gas in a closed vessel. We should find that the pressure is doubled since the volume has to remain constant. The constant K is equal to the mass of gas multiplied by a new constant K', where $K = mK'$.

$$\therefore pV = \text{mass of gas} \times \text{a constant } K'$$

Therefore $pV = mK'$, where K' depends only on the type of gas used (oxygen, nitrogen, etc.) and the temperature. Therefore $p = \frac{m}{V} \times K'$, but the mass of gas divided by its volume is its density, ρ. Therefore $p = \rho \times K'$. So that *Boyle's Law* may be stated in another way: THE PRESSURE OF A GAS IS PROPORTIONAL TO ITS DENSITY WHEN THE TEMPERATURE IS CONSTANT.

IT HAS been said: "All roads lead back to Greece"; this time to Archimedes, an eminent mathematician who lived about 200 B.C. You must all know the story of how he discovered his principle of buoyancy. ARCHIMEDES' PRINCIPLE states that THE APPARENT LOSS IN WEIGHT OF A SOLID WHEN IMMERSED IN A FLUID IS EQUAL TO THE WEIGHT OF THE FLUID DISPLACED. Rather than tell the "bath-tub story" we shall give a formal proof of this principle:

Suppose a solid cylinder, whose cross-section is A sq.cm. in area, whose height is h, is immersed in a liquid of density ρ. The volume of the cylinder is hA cc. The top of the cylinder is h_1 cm. below the surface of the liquid and the base of the cylinder h_2 cm. below.



$$\therefore\ h = h_2 - h_1\ .$$

The pressure of the liquid on the top of the cylinder is $h_1 g\rho$ dynes per sq.cm. Therefore the downward force on the top of the cylinder is $h_1 g\rho A$ dynes.

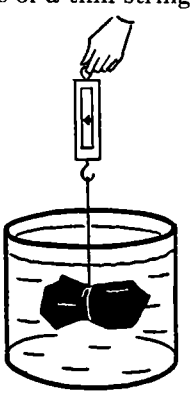
The pressure of the liquid at the bottom of the cylinder is $h_2 g\rho$ dynes per sq.cm. The upward force on the bottom of the cylinder is $h_2 g\rho A$ dynes.

$$\therefore \text{ Total upward vertical force on cylinder} = h_2 g\rho A - h_1 g\rho A$$
$$= g\rho A(h_2 - h_1) = g\rho Ah \text{ dynes}$$

The volume of the liquid displaced by the cylinder is the same as the volume of the cylinder, which is Ah cc. The mass of this amount of liquid is $Ah\rho$ gm. and its weight $Ah\rho g$ dynes. Thus we have shown that the **force pushing the cylinder up, $Ah\rho g$ dynes, is equal to the weight of the displaced fluid.** The cylinder weighs less in the fluid than it does in air because of this buoyant force. Does this buoyant force depend on how deep the cylinder is immersed below the surface?§ Though this result has been proved only for a cylinder, it is true for an object of any shape immersed in a fluid.

For an object to float the weight of the object must be less than the weight of fluid it displaces when completely immersed. Does this mean that the density of the fluid must be greater than the density of the object?§§ When an object floats the weight of the displaced fluid equals the weight of the object.

EXAMPLE 1. A piece of iron whose mass is 50 gm. is hung in oil by means of a thin string attached to a spring balance. The oil has a density of 0.8 gm. per cc. Find the reading of the spring balance. The density of the iron is 7.8 gm. per cc.

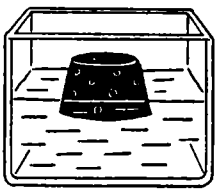


$$\text{Density} = \frac{\text{mass}}{\text{volume}} \qquad \therefore\ \text{Volume} = \frac{\text{mass}}{\text{density}}$$

$$\therefore \text{ Volume of 50-gm. piece of iron} = \frac{50}{7.8} = 6.4 \text{ cc.}$$

$$\text{Mass of oil displaced} = \text{volume of iron} \times \text{density of oil}$$
$$= 6.4 \times 0.8 = 5.1 \text{ gm.}$$
$$\text{Weight of oil displaced} = 5.1 \text{ gm. wt.}$$
$$= \text{apparent loss in weight of iron}$$
$$\therefore \text{ Spring balance reads } 50 - 5.1 = 44.9 \text{ gm. wt.}$$

EXAMPLE 2. A cork whose volume is 10 cc. is floating on water. Find how many cc. of the cork are above water.



Density of cork = 0.25 gm. per cc.; Density of water = 1 gm. per cc.

Mass of cork = volume × density = $10 \times 0.25 = 2.5$ gm.

For the cork to float a mass of water weighing 2.5 gm. must be displaced.

$$\text{Thus volume of water displaced} = \frac{\text{mass}}{\text{density}} = \frac{2.5}{1} = 2.5 \text{ cc.}$$

Hence (10 − 2.5) or 7.5 cc. of the cork are above the surface of the water.

A DISCOVERY of considerable importance was made when a mercury barometer was carried up a mountain and it was found that the height of the mercury column decreased the higher it was carried. Atmospheric pressure decreases with elevation.

Have you ever thought about how balloons manage to rise? The problem is an application of Archimedes' principle. A large bag displaces a large quantity of air. Thus there is a buoyant force equal to the weight of the air displaced. But the bag must be inflated with some gas and unless this gas is lighter than air the buoyant force will be counterbalanced by the weight of the gas in the bag.

EXAMPLE 3. Let us find what the buoyant force is on a bag whose volume is 1.6×10^9 cc. (about 60,000 cu.ft.) and which is filled with hydrogen. The weight of the bag itself is 2×10^5 gm. wt. The density of air we may take as 0.00129 gm. per cc., and the density of hydrogen as 0.00009 gm. per cc.

Mass of air displaced = volume × density = $1.6 \times 10^9 \times 0.00129$ gm.

This produces a buoyant force on the bag of this amount.

Mass of hydrogen in bag = volume × density = $1.6 \times 10^9 \times 0.00009$ gm.

Weight of hydrogen in bag = $1.6 \times 10^9 \times 0.00009$ gm. wt.

The buoyant force lifts the bag up while the weight of the bag and the hydrogen pulls it down. Thus

$$\begin{aligned}\text{Resultant lifting force} &= \text{buoyant force} - \text{wt. of hydrogen} - \text{wt. of bag} \\ &= (1.6 \times 10^9 \times .00129) - (1.6 \times 10^9 \times .00009) - (2 \times 10^5) \\ &= 1.6 \times 10^9 \times (.00129 - .00009) - (2 \times 10^5) \\ &= (1.6 \times 10^9 \times .00120) - (2 \times 10^5) \\ &= (1.92 \times 10^6) - (2 \times 10^5) = 1.72 \times 10^6 \text{ gm. wt.}\end{aligned}$$

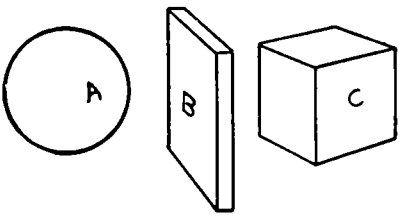
Notice that the resultant force apart from the weight of the bag is equal to the (volume of the bag) × (density of air − density of hydrogen).

§ Answer to question on the previous page: No.

§§ Answer to question on the previous page: Yes.

PROBLEMS

1. Three vessels, *A*, *B*, and *C*, are all made of the same material and all have the same total surface area.

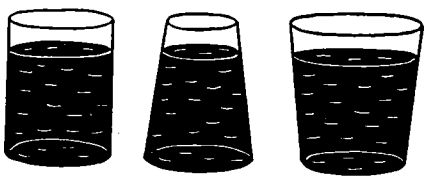


a) If they are all evacuated, which would cave in first? second? last?

b) If they are all airtight and are sunk to a considerable depth in a lake, which would collapse first? second? last?

2. On a very humid day when the air contains a lot of water vapor, will the barometer stand higher or lower than on a day when the air is dry? (The density of a mixture of air and water vapor is less than that of dry air.)

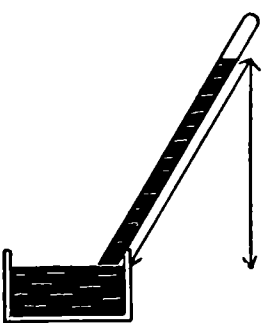
3. A paradox, a hydrostatic paradox! Can you solve it? Three vessels, as shown in the diagram, are all filled to the same height with the same liquid. The area of the base of each vessel is the same.



a) Is the pressure on the base of each the same?

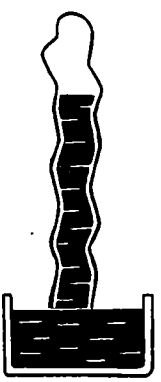
b) Is the total force on each base the same?

c) If the bases were all loose, would it require the same vertical force on each to hold in the liquid?



4. If a mercury barometer is tilted, will the barometric height be measured by the length of the mercury column, or by the vertical height of the top of the mercury from the well of mercury?

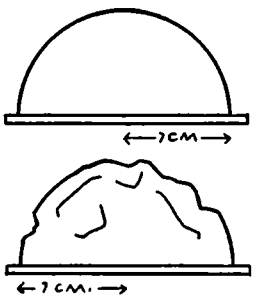
5. If the pressure on a given mass of gas is doubled, how is its volume affected? The temperature remains constant.



6. A toy balloon is filled with hydrogen at a pressure of 70 cm. of mercury. Its volume is 1,000 cc. If the outside air pressure increases to 80 cm., what is the volume of the balloon? The temperature is assumed to be constant.

7. If a mercury barometer is made of crooked tubing, but is otherwise properly made, will the barometer read correctly?

8. A hemispherical shell with a diameter of 14 cm. is placed on a flat glass plate in such a way that the connection is airtight. Find the force required to separate the hemispherical shell from the glass plate when the shell is

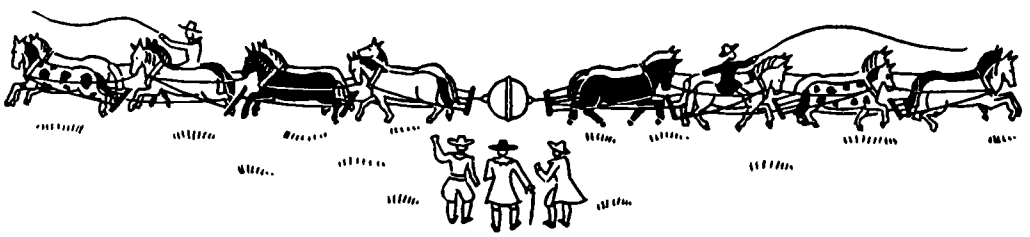


a) Completely evacuated. (Pressure of atmosphere 76 cm. of mercury = 1.01×10^6 dynes per sq.cm.)

b) Filled with air at half an atmosphere.

If the base of the shell is not touched but the shell is badly dented,

c) What force will be required to separate the two after the shell is again evacuated?



9. Otto von Guericke (1602–86), the burgomaster of Magdeburg, fixed two copper hemispheres 42 cm. in diameter tightly together by means of a greased leather washer. He then exhausted the air through a side tube so that the hemispheres became tightly pressed together as a result of atmospheric pressure. Guericke used 8 horses attached to each hemisphere, or 16 horses in all, to separate the two hemispheres.

a) Find the force holding the two hemispheres together. (Take $\pi = \frac{22}{7}$.)

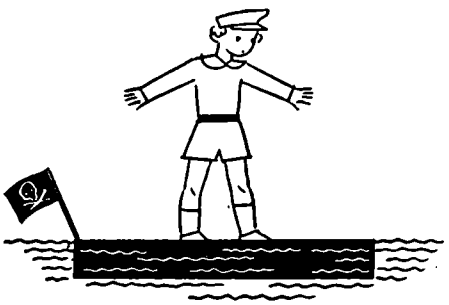
b) Would the effect (apart from making a great impression) have been the same had he tied one of the hemispheres to a tree and used only 8 horses?

10. The icebergs seen in plenty in the North Atlantic about June are a great menace to navigation. If the density of salt water is 1.025 gm. per cc. and the density of an iceberg is 0.90 gm. per cc., find what proportion of the total volume of an iceberg is below the water line.

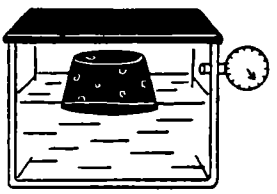
11. King Hiero of Syracuse asked Archimedes to find out if his supposedly golden crown contained any silver. It was this problem that led Archimedes to the discovery of his principle of buoyancy. Obviously King Hiero did not wish his crown spoiled. The crown weighed 10,000 gm. wt. in air and 9,373 gm. wt. in water. The density of gold is 19.3 gm. per cc. and that of silver is 10.5 gm. per cc.

a) Was King Hiero cheated?
b) What is the volume of the crown?
c) If so how many gm. of silver did the crown contain?

12. A boy whose mass is 50,000 gm. builds himself a raft out of wood with a density of 0.7 gm. per cc. He is short of wood, so he uses just enough to keep his toes out of water when he stands on the raft. Find the mass of the raft.



13. A cork is floating on water in an airtight vessel. If the pressure of the air in the vessel is increased, will more, less, or the same amount of the cork be under water?



14. If water were used in place of mercury as the liquid in a barometer tube, find how high (in cm. and ft.) the water column would stand on a day when the pressure is 76 cm. of mercury. (Density of mercury = 13.6 gm. per cc.)

15. A bubble of marsh gas, methane, has a volume of 0.1 cc. when it is liberated by chemical means from the bottom of a pond 5 m. deep. Find the volume of the bubble when it reaches the top of the pond.

16. Normal atmospheric pressure is about 10^6 dynes per sq.cm. This is equivalent to a weight of about 1 kg. acting on every sq.cm. of your body. How is it you do not collapse?

17. The weights used for ordinary balances are made of brass which has a density of 8.5 gm. per cc. These weights are subject to a buoyant force due to the weight of air which they displace. In the case of a 100 gm. wt. how much is this buoyant force? (Density of air = 0.00129 gm. per cc.)

18. A barometer is made of glass tubing 1 sq.cm. in internal cross-section. The mercury rises to 75 cm. in the tube, leaving 25 cm. above the mercury as a vacuum. A bubble of air is introduced into the vacuum and the mercury drops to 70 cm.
a) What is the pressure and volume of the air bubble?
b) What is the volume of this air bubble at an atmospheric pressure of 75 cm.?

c) If the density of air at this pressure and temperature is 0.0012 gm. per cc., find the weight of this bubble of air.

19. A cubical block of mahogany, 10 cm. on an edge, is floating in water.
   a) If its density is 0.85 gm. per cc., what is the volume of the mahogany above water?
   b) The mahogany floats in olive oil having a density of 0.92 gm. per cc. What volume of the mahogany is above the oil?

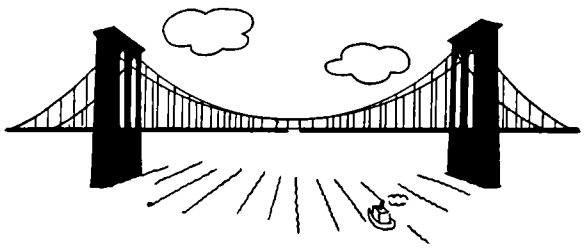
20. The spherical gondola used in a stratosphere flight was hermetically sealed and the pressure inside was kept at about 70 cm. of mercury. What was the total force pushing outward on the gondola when the outside pressure was 5 cm. of mercury? The diameter of the gondola was 2 m. (The surface area of a sphere of diameter d is πd^2.)

21. In the Settle-Fordney stratosphere flight a balloon was used which had a capacity of approximately 1.6×10^{10} cc. at a pressure of 75 cm. *Hg*. The balloon was filled to one-fifth of its capacity with pure hydrogen (density 9×10^{-5} gm. per cc.). The balloon had a weight of 9×10^{5} gm. and the gondola and men a weight of 2×10^{5} gm. (Density of air = .00129 gm. per cc.)
   a) What was the greatest weight (scientific instruments and ballast) which could be added and still allow the balloon to rise?

   The bottom of the balloon was open, so that when the balloon got to its maximum altitude, corresponding to a pressure of 5 cm. of mercury, the excess hydrogen could escape (the bag now being full).
   b) How much weight could the balloon lift at this maximum altitude?
   c) Therefore how much ballast in the form of fine lead shot had been thrown out on the way up? (Assume that the temperature of the hydrogen in the bag and that of the outside air is always the same.)

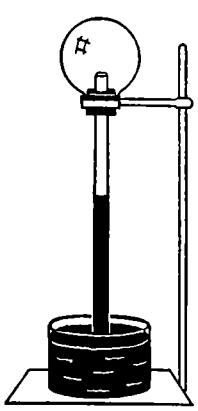
22. A rectangular zinc bath tub has a base 200 cm. long and 50 cm. broad. The height of the tub is 50 cm. and it is filled with water to a height of 30 cm.
   a) Find the force due to the water pressure on the sides and ends of the tub.

   When a boy floats in the tub, the water-level rises to a height of 34.5 cm.
   b) Find the weight of the boy.
   c) If the boy is nine-tenths submerged, what is his density?



TEMPERATURE AND EXPANSION

THE SENSE of hot and cold develops quite early in life after a few experiences with hot stoves or irons. We are able to tell very roughly by the sense of touch whether one object is hot and another cold or whether one object is warmer than another. Now when objects are heated some of their physical properties such as length, volume, density, electrical resistance, etc., are changed. It is possible to measure temperature by the change in any one of these properties. Let us first examine how we define temperature. If you stand under a hot shower you sense the feeling of warmth through the fact that heat is transferred or conducted from the hot water to your body. On the other hand, a shivery cold shower conducts heat away from your body. Thus, if we have two objects in contact, heat flows from the one at the higher temperature to the one at the lower temperature. IT IS THIS PHYSICAL CONDITION CALLED TEMPERATURE WHICH DETERMINES THE DIRECTION OF FLOW OF HEAT BETWEEN TWO OBJECTS PLACED IN CONTACT. Of all the physical properties which change with temperature, the change in volume is the one most commonly used as the temperature measuring device or thermometer.

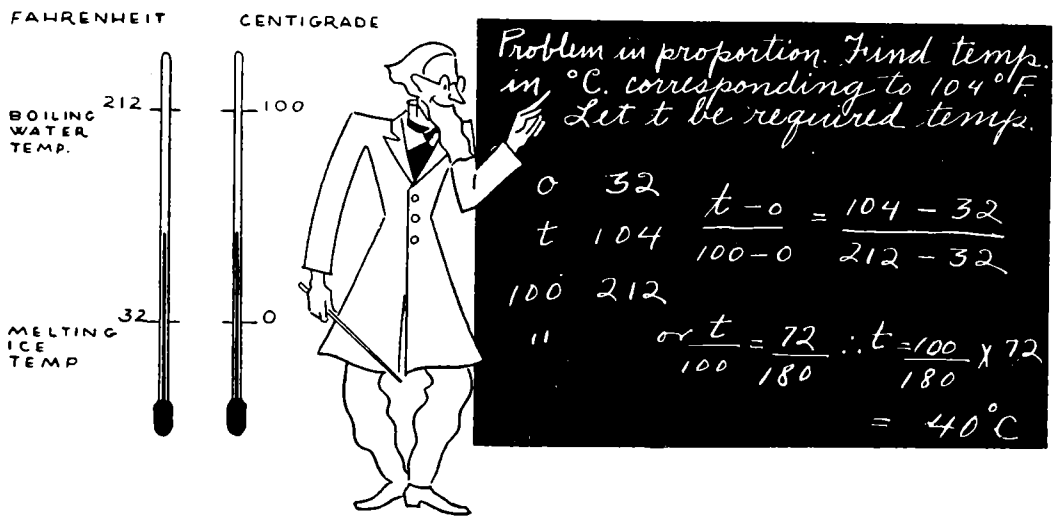


ONE OF THE earliest thermometers was made by Galileo, who made use of the expansion of air. The increase in volume of air resulting from an increase in temperature forced water down a tube. This instrument responds not only to changes in temperature but also to changes in atmospheric pressure, so it is not a satisfactory thermometer. Liquids and gases expand very much more than do solids for a given increase in temperature. A liquid in a glass container is the most usual type of thermometer. A small bulb is blown on the end of a glass capil-

lary tube. Some liquid is then introduced into the bulb by forcing out the air with heat. After the appropriate amount of liquid has been introduced, the end of the capillary is sealed off. The liquid is usually alcohol or mercury, depending on the temperature range to be measured.

Having made our thermometer, we must next decide on the unit or scale of temperature. There are two scales in common use, the Centigrade and the Fahrenheit. The usual calibration temperatures are the melting point of ice and the boiling point of water at an atmospheric pressure of 76 cm. of mercury.

COMPARISON OF THE TWO TEMPERATURE SCALES


FAHRENHEIT
CENTIGRADE
BOILING WATER TEMP.
212
100
MELTING ICE TEMP
32
0
Problem in proportion. Find temp. in °C. corresponding to 104 °F. Let t be required temp.
0 32
t 104
100 212
t − 0 / 100 − 0 = 104 − 32 / 212 − 32
or t/100 = 72/180 ∴ t = 100/180 × 72
= 40°C

On the Fahrenheit scale 104° (104° F.) represents the same temperature as 40° on the Centigrade scale (40° C.).

EXPANSION OF SOLIDS: We are all familiar with the fact that objects increase in length with temperature. A long steel bridge is not rigidly fixed at both ends because provision must be made for its expansion or contraction with temperature. If the length of a solid bar is 1 cm. at 0° C. and we warm it up until its temperature is 1° C., the increase in length is called its COEFFICIENT OF LINEAR EXPANSION and is designated by a. If we take a solid bar 100 cm. long at 0° C. and warm it up to 1° C., it will increase its length one hundred times as much as would a bar 1 cm. long. Also if we warm the solid bar, which has a length of 100 cm. at 0° C., to 10° C., the expansion of this bar would be one thousand times that of a bar 1 cm. long warmed from 0° C. to 1° C. If a is the coefficient of linear expansion of a bar, then a bar l_0 cm. long at 0° C. would increase its length by l_0at when heated

to $t°$ C. Thus if l_t is the length of the bar at $t°$ C., its increase in length is $l_t - l_0$, so that $l_t - l_0 = l_0 \alpha t$.

EXAMPLE. Find the length of Brooklyn Bridge, approximately 1,800 m. long at 0° C., when the temperature increases to 40° C. on a hot summer's day. The coefficient of linear expansion of steel, α, is 0.000012, i.e., 1.2×10^{-5}, per °C.

Increase in length:

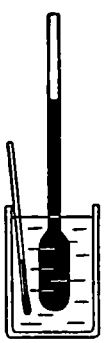
$$l_t - l_0 = l_0 \alpha t;\ l_0 = 1{,}800 \times 100 = 1.8 \times 10^5 \text{ cm.};\ t = 40° \text{ C.}$$

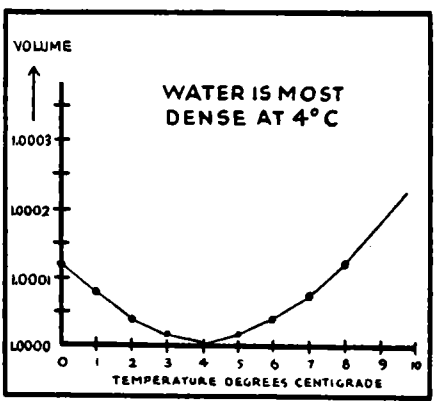
$$l_{40} - l_0 = l_0 \alpha t = 1.8 \times 10^5 \times 1.2 \times 10^{-5} \times 40 = 86.4 \text{ cm.}$$

Thus the length of the bridge at 40° C. is 86.4 cm. greater than at 0° C., i.e., 180,086.4 cm.

Had we not been given the length of the bridge at 0° C. but at some other temperature, as, for instance, 10° C., and been asked to find its length at 50° C., we could say without any appreciable loss in accuracy that $l_{50} - l_{10} = l_{10} \alpha (50 - 10)$. That is, the increase in length is the same for a temperature rise from 10° C. to 50° C. as it is for a rise from 0° C. to 40° C. This is true *only* because α is very small for all solids. If l_{t_2} represents the length of a bar at $t_2°$ C. and l_{t_1} the length at some lower temperature $t_1°$ C., then $l_{t_2} - l_{t_1} = l_{t_1} \alpha (t_2 - t_1)$.

EXPANSION OF LIQUIDS: One of the simplest means of determining the expansion of a liquid is shown in the diagram. The liquid must be held in a vessel. When the temperature is raised the vessel as well as the liquid will expand. The liquid will rise in the narrow vertical tube, but this increase in volume will be the *difference* between the volume increase of the liquid and that of the vessel. However, the expansion coefficient of most liquids is of the order of one hundred times that of solids. Now, of course, we must speak of a *volume coefficient* of expansion.



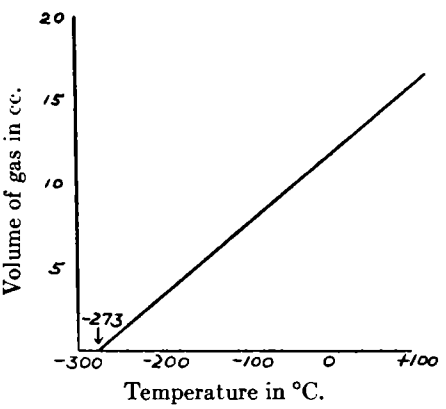


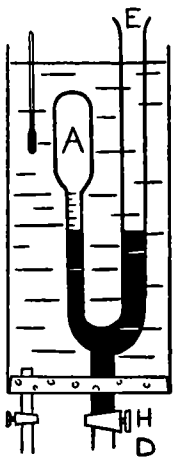
Liquids in general do not expand uniformly. The most interesting example of this is water. Water has a minimum volume at 4° C., and consequently a maximum density at 4° C. This is of enormous importance in nature.

EXPANSION OF GASES: Different solids and liquids have different coefficients of expansion. Gases are much more orderly in this respect: They *all* behave alike except for minor differences which we shall neglect.

We have already learned that an increase of pressure changes the volume of a gas when the temperature is constant, so when we allow a gas to expand by increasing its temperature we must be careful about the pressure. Let us first determine how a gas expands with temperature when the pressure

remains constant. The *coefficient of volume expansion of a gas at constant pressure* may be found with the apparatus shown in the diagram. The gas is contained in the vessel A, which is sealed at the lower end by a mercury column. The temperature of the gas is changed by changing the temperature of the water surrounding the inside apparatus. In order to keep a constant pressure on the gas, the level of the mercury in the two arms must be kept the same. To do this mercury must be poured in at E or let out at D according to whether the temperature is increased or decreased. The tube A is graduated so that the different volumes occupied by the gas at different temperatures can be measured. What is the constant pressure used in this experiment?§ The results of this experiment can be expressed in the form of CHARLES'S LAW: THE VOLUME OF A GIVEN MASS OF GAS KEPT AT CONSTANT PRESSURE INCREASES BY $\frac{1}{273}$ OF ITS VOLUME AT 0° C. FOR EACH DEGREE CENTIGRADE RISE IN TEMPERATURE. Thus if V_t represents the volume of the gas at $t°$ C. and V_0 its volume at 0° C., then the increase in volume $V_t - V_0$ is equal to $V_0 \times \frac{1}{273} \times t$.





EXAMPLE. Find the volume which 10 cc. of hydrogen at 0° C. occupies at 20° C. The pressure remains constant.

$$V_t - V_0 = V_0 \times \frac{1}{273} \times t$$

$$V_t - 10 = 10 \times \frac{1}{273} \times 20 = 0.73$$

$$V_t = 10.73 \text{ cc.}$$

We may re-write

$$V_t = V_0 + (V_0 \times \frac{1}{273} \times t) \text{ as } V_t = V_0\left(1 + \frac{t}{273}\right)$$

$$V_t = V_0\left(\frac{273+t}{273}\right)$$

We shall now define a new temperature scale which has the same unit as the Centigrade scale but whose zero is at $-273°$ C. This new scale is called the ABSOLUTE TEMPERATURE (or KELVIN) SCALE. The zero of the Centigrade

scale is 273° A. Ten degrees Centigrade is 283° A. We shall use T to denote temperatures on the absolute scale and t for temperatures on the Centigrade scale: $273+t=T$. Thus

$$V_t=\frac{V_0(273+t)}{273}=\frac{V_0T}{T_0}\ ; \qquad \frac{V_T}{V_0}=\frac{T}{T_0} \text{ or } \frac{V_T}{T}=\frac{V_0}{T_0}$$

This result states that THE VOLUME OF A GIVEN MASS OF A GAS IS DIRECTLY PROPORTIONAL TO ITS ABSOLUTE TEMPERATURE IF THE PRESSURE IS CONSTANT. V is proportional to T; $V/T=$ a constant. The volume of the 10 cc. of hydrogen of the previous problem can now be determined at 20° C. (i.e., 293° A.) in a very simple manner:

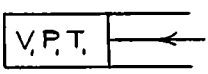
$$V_{293°A}=\frac{V_0T}{T_0}=10\times\frac{293}{273}=10.73 \text{ cc.}$$

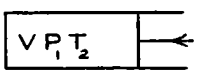
WE NOW have two laws which a gas obeys:* BOYLE'S LAW: at constant temperature, $pV=$ a constant; CHARLES'S LAW: at constant pressure, $V/T=$ a constant. These two laws may be combined into a single one to give the GENERAL GAS LAW: $pV/T=$ a constant.†

§ Answer to question on the previous page: Atmospheric pressure.

* A gas which obeys these two laws is called *perfect gas.*

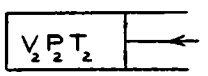
† There is no necessity to read this footnote, but if you have sufficient curiosity and patience you may like to see how Boyle's and Charles's laws are combined to give the general gas law. To do this let us imagine we have a gas contained in a cylinder closed at one end by a movable, frictionless airtight piston. The volume of the gas is V_1 when the pressure is p_1, and its absolute temperature is T_1. The temperature of the gas is increased to some new value T_2 and as a result the volume increases to V; the pressure during this is kept constant at p_1. By Charles's law:





$$\frac{V}{T_2}=\frac{V_1}{T_1} \text{ or } V=\frac{V_1T_2}{T_1}=V_1\times\frac{T_2}{T_1}$$

The pressure on the gas is now increased to p_2 and as a result the volume decreases from V to some value V_2. The temperature during this experiment is kept constant at T_2. We can apply Boyle's law to this change:



$$p_2V_2=p_1V \text{ or } V=\frac{p_2}{p_1}\times V_2$$

Thus:

$$V=\frac{p_2V_2}{p_1}=\frac{V_1T_2}{T_1} \text{ or } \frac{p_2V_2}{T_2}=\frac{p_1V_1}{T_1}$$

The gas originally occupied a volume V_1 when its pressure was p_1 and temperature T_1. Without allowing any gas to escape or enter the apparatus, the pressure, volume, and temperature of the gas were changed to some new values, p_2, V_2, and T_2, and we found that pV/T for gas in the original state $=pV/T$ for gas in the new state. (The apparatus used for proving Charles's law can also be used to go through the operations that have been outlined for this cylinder and piston.)

FOR A GIVEN MASS OF GAS THE PRODUCT OF ITS PRESSURE AND ITS VOLUME DIVIDED BY ITS ABSOLUTE TEMPERATURE IS A CONSTANT.

EXAMPLE. A given mass of gas occupies a volume of 100 cc. at a temperature of 20° C. and a pressure of 70 cm. of mercury. Find the volume of the gas when its temperature is 40° C. and the pressure on it is 76 cm. of mercury. General gas law pV/T, or, stated in another manner:

$$\frac{p_1V_1}{T_1}=\frac{p_2V_2}{T_2} \text{ or } \frac{V_2}{V_1}=\frac{T_2}{T_1}\times\frac{p_1}{p_2}. \qquad V_2=V_1\times\frac{T_2}{T_1}\times\frac{p_1}{p_2}$$

$V_1=100$ cc. $p_1=70$ cm. of mercury $T_1=273+20=293°$ A.

$V_2=?$ $p_2=76$ cm. of mercury $T_2=273+40=313°$ A.

$$V_2=100\times\frac{313}{293}\times\frac{70}{76}=98.4 \text{ cc.}$$

The gas, then, has a volume of 98.4 cc. when its temperature is 40° C. and its pressure 76 cm. of mercury.

YOU MIGHT wonder why we have not discussed how the pressure of a gas changes with temperature when the volume remains constant. Actually we have already done this when we combined Boyle's and Charles's laws to give the general gas law:

$$\frac{p_1V_1}{T_1}=\frac{p_2V_2}{T_2}$$

The pressure of the gas increases with temperature. If the volume is kept constant, we may put $V_1=V_2$ in the foregoing equation. We then have

$$\frac{p_1}{T_1}=\frac{p_2}{T_2}, \qquad \text{i.e.,} \qquad \frac{p}{T}=\text{a constant}$$

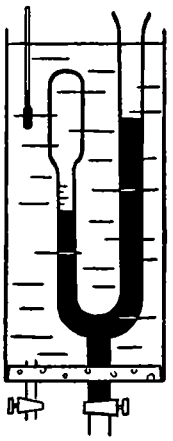
This is a similar equation to that of Charles's law where the pressure remains constant:

$$\frac{V_1}{T_1}=\frac{V_2}{T_2}, \qquad \text{i.e.,} \qquad \frac{V}{T}=\text{a constant}$$

So that the pressure of a gas is proportional to its absolute temperature when the volume remains constant, just as the volume of a gas is proportional to its absolute temperature when the pressure remains constant. Another way of saying this is that the coefficient of pressure increase with temperature for a gas kept at constant volume is $\frac{1}{273}$, i.e.,

$$p_t=p_0+p_0\times\frac{1}{273}\times t.$$

This result can be obtained experimentally with the same apparatus used for finding the volume increase of a gas with temperature at constant pressure. In the present case the volume is kept constant by adjusting the mercury column; the pressure is atmospheric pressure plus or minus the difference in height of the two columns.



COLLECTED RESULTS OF SUCH AN EXPERIMENT ON A DAY WHEN THE ATMOSPHERIC PRESSURE IS 75 CM. OF MERCURY

Expt.	Pressure in Cm. of Hg	Temp. °C.	Temp. °A.	$\frac{\text{Pressure}}{\text{Temp. °A.}}$
1	75+6=81	24	273+24=297	$\frac{81}{297}=\frac{3}{11}$
2	75	2	273+2=275	$\frac{75}{275}=\frac{3}{11}$
3	75−3=72	− 9	273−9=264	$\frac{72}{264}=\frac{3}{11}$

It is convenient to adopt some **standard temperature and pressure (STP). The conditions are 0° C. and 76 cm. of mercury.**

PROBLEMS

1. Can you suggest anything from your experience to support the fact that water expands on freezing?

2. Why is it that hot air rises?

3. The pressure of air in an automobile tire is 4 atmospheres when its temperature is 7° C. After traveling some distance the tire and air inside become heated to 27° C. What is the pressure of the air in the tire now, assuming that the tire does not stretch?

4. A liter of hydrogen at STP has a mass of 0.0898 gm.
   a) What is the density of hydrogen at STP?
   b) Find the volume of this liter of hydrogen when the pressure increases to 84 cm. and the temperature to 45.5° C.
   c) Find the density of the hydrogen under these conditions.

5. *a*) Find to the nearest whole number the mass of 22.4 liters of hydrogen at STP.

   The mass of a liter of nitrogen at STP is 1.25 gm.
   b) What volume does 28 gm. of nitrogen occupy at STP?
   c) Calculate the value of pV/T for 28 gm. of nitrogen at STP.
   d) Calculate the value of pV/T for 2 gm. of hydrogen at STP.

6. Out of doors on a chilly zero day (0° C.) a barometer reads 74.25 cm. of mercury. The barometer is brought inside to a room where the temperature is 35° C.
   a) Is the air pressure inside the house the same as that outside?
   b) Will the barometer read the same inside the house as outside?
   c) Find what the barometer will read in the room if the density of mercury at 0° C. is 13.6 gm. per cc. and at 35° C. is 13.5 gm. per cc.

7. A steel tape which is exactly 10 m. long at 0° C. is used on a summer's day when the temperature is 30° C. Find how much the tape is in error at this temperature. (The linear coefficient of expansion of steel is 1.2×10^{-5} per °C.)

8. The Golden Gate Bridge in California is the largest single arch suspension bridge in the world. If the length of this bridge is 1.25×10^{5} cm. at 5° C., find its increase in length on a day when the temperature is 35° C.

9. The steel platform of a weighing machine is square. At 5° C. each side measures 100 cm. What is the increase in area of this platform at 55° C.? (First find the length of each side at 55° C. and then compute the new area.)

10. A cubical block of steel, 100 cm. on an edge at 0° C., is heated to 50° C. Find the increase in volume.

11. Pyrex glass has a lower coefficient of expansion than soda glass. Boiling hot water is poured into a vessel made of pyrex and into a similar one made of soda glass. Which is more liable to crack?

12. If the density of steel is 7.80 gm. per cc. at 0° C., is its density at 50° C. greater or less than 7.80 gm. per cc.?

13. Thermometers usually contain either alcohol or mercury.
   a) From the following data, which liquid would be used in a thermometer for measuring high temperatures?
   b) In one for measuring low temperatures?

	Alcohol	Mercury
Boiling point	78° C.	357° C.
Freezing point	−130	−39

c) What would these temperatures be on the Fahrenheit scale?

d) Perhaps you would rather find out what $-40°$ F. would be on the Centigrade scale. Frequently a formula, $°C. = \frac{5}{9}(°F. - 32)$, is used to convert from degrees Fahrenheit to degrees Centigrade. Show that using this formula would give the same results as the method given on the blackboard (p. 81).

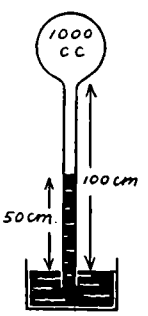
14. Find the total mass of air in a lecture room 20 m. by 15 m. by 5 m. when the temperature is 27° C. and the pressure is 72 cm. of mercury. The density of air at STP is 0.00129 gm. per cc.

15. This is a hypothetical problem. A liquid with a small coefficient of expansion is placed in a vessel made of something with a larger coefficient of expansion. (Water in a paraffin tube does this for a small temperature range.)

a) When the liquid and vessel are heated, will the liquid rise or fall in the tube?

b) When cooled what happens?

16. An air thermometer similar to Galileo's was made by sealing a long glass tube 1 sq.cm. in cross-section to a liter flask. A slight amount of air is driven out by warming the flask so that, when the end of the tube is dipped under water, water rises in the tube. Suppose that at a temperature of 21° C. and an atmospheric pressure of 78 cm. of mercury the water rises 50 cm. in the tube and that the length of the tube is 100 cm.



a) If the atmospheric pressure remains constant but the temperature falls to 7° C., how high does the water-level rise?

b) If now the temperature remains at 7° C. but atmospheric pressure changes from 78 to 75 cm. of mercury, find the height of the water column. (Neglect the pressure due to the water in the tube.)

17. When freezing weather sets in

a) Does the whole of the water in a pond cool off to 4° C. before any freezes?

b) Does it all cool off to 2° C. before any freezes?

c) When the upper layer cools below 4° C. does this upper layer of colder water sink to the bottom of the pond?

d) Will a shallow pond become safe for skating sooner than a deep pond under the same conditions?

e) Give a reason for your answer in (*d*).

18. A piece of wood is floating in water at 5° C. Would more, less, or the same amount of the wood be out of water if the temperature of the water were increased to 20° C.?

19. A toy balloon is filled with 2,000 cc. of hydrogen gas at atmospheric pressure and at a temperature of 27° C. Find the volume of the balloon if the temperature changes to 0° C., assuming the pressure remains atmospheric.

20. A closed vessel contains 22.4 liters of hydrogen at STP.

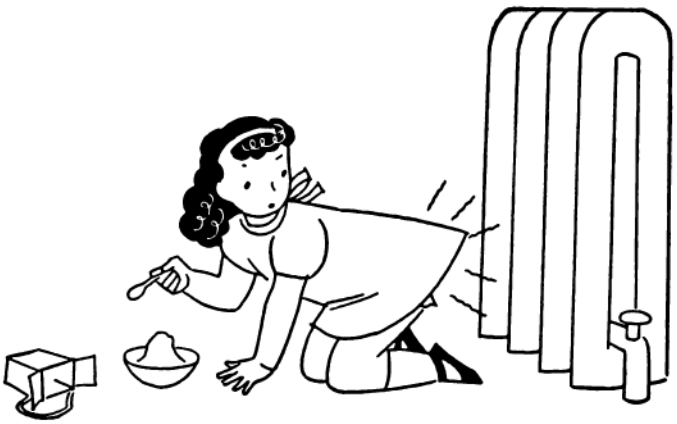
a) What is the mass of the hydrogen in the vessel?
If the temperature of the hydrogen is raised to 91° C.?

b) Find the pressure exerted by the hydrogen, assuming the volume of the vessel remains constant.

21. Gas is contained in a vessel similar to the one shown in the diagram on page 83.
The temperature of the gas is 0° C., the pressure 76 cm. of mercury, and the volume it occupies is 500 cc.

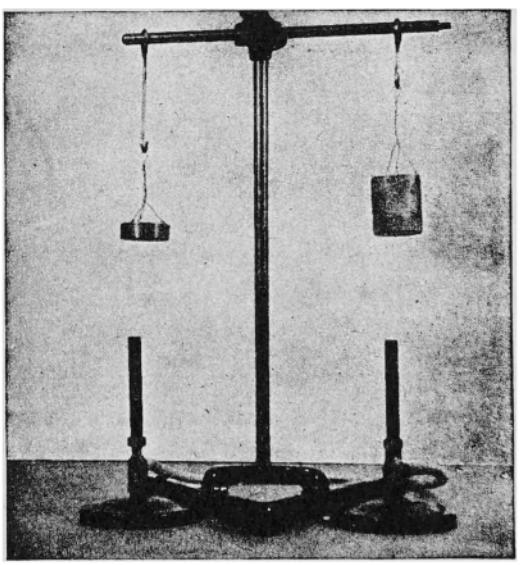
a) Find the volume of the gas when the pressure changes to 57 cm. of mercury and the temperature to 45.5° C.

b) Find the volume of the gas when the pressure changes to 95 cm. of mercury and the temperature to −45.5° C.



HEAT

UP TO the present we have raised the temperature of a substance but have not considered how much heat must be used to do this. Different substances require different amounts of heat to produce the same rise in temperature even when their masses are the same: Suppose we supply the same quantity of heat to 10 gm. of aluminum and to 10 gm. of water.* We then adjust two Bunsen burners so that they look as nearly as possible alike and put one under the aluminum and the other under the water for the same length of time. When the water is still cool enough to dip a finger in, the aluminum is too hot to touch. Supplying the same amount of heat to equal masses of aluminum and water has increased the temperature of the aluminum very much more than that of the water. Actually the increase in temperature of the aluminum is about five times that of the water. If we wish to raise the temperature of equal masses of aluminum and water by



* The water will have to be contained in a vessel but this we can make so thin that in this rough experiment it may be neglected.

the same amount, the water would require about five times the quantity of heat that the aluminum would. Now we must decide on a unit to use in measuring quantity of heat. The unit used is the *calorie.* **One calorie is the amount of heat required to raise the temperature of 1 gm. of water 1° C.**

IF WE had chosen 1 gm. of aluminum and 1 gm. of water for our previous experiment and had left the Bunsen burners under until the temperature of the water had increased 1° C., the temperature of the aluminum would have increased by 5° C. One calorie of heat raises the temperature of 1 gm. of aluminum 5° C., *not* 1° C. as in the case of water. Only $\frac{1}{5}$ cal. of heat would be required to raise the temperature of 1 gm. of aluminum 1° C. **The number of calories of heat required to raise the temperature of 1 gm. of a substance 1° C. is known as the specific heat of the substance.** Thus the specific heat of aluminum is $\frac{1}{5}$ while that of water is 1†. The number of calories required to raise the temperature of 10 gm. of aluminum 15° C. will be $10 \times \frac{1}{5} \times 15$, i.e., 30 cal. The number of calories required to raise the temperature of m gm. of a substance of specific heat s from t_1° C. to t_2° C. is equal to the *mass* × *specific heat* × *rise in temperature:*

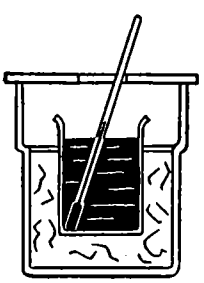
$$\text{Quantity of heat} = m \times s(t_2 - t_1)$$

The experiment we performed with the aluminum is far too rough to give accurate results for the specific heat, so we must look for some better method.

DETERMINATION OF specific heat by the method of mixtures. A known mass of a substance is heated up to some temperature and then quickly transferred to a vessel containing cold water. The mass and temperature of the vessel and water are known. The hot substance cools down; the cold water and container warm up. If we take special precautions to prevent any heat escaping to the outside (which is almost impossible):

Heat lost by hot substance = heat gained by (cold water + container)

This principle is the basis of calorimetry (i.e., heat measurement). The vessel which contains the cold water is given the special name, *calorimeter.* In order to minimize the amount of heat escaping from the calorimeter it is placed on an insulating support inside another vessel. Loosely packed cotton, a good heat insulator, is placed in the space between the two vessels.



† Specific heat is often defined as the ratio of the amount of heat required to raise 1 gm. of a substance 1° C. to that required to raise 1 gm. of water 1° C. In this case specific heat would have no units.

EXAMPLE. The specific heat of silver may be found by this method: Suppose a silver block whose mass is 200 gm. is heated to 100° C. and is then quickly transferred to the calorimeter which contains 250 gm. of water at 18° C. The calorimeter made of copper (specific heat 0.09) has a mass of 100 gm. The temperature of the water and calorimeter is increased to 21.4° C. as a result of placing the hot silver object in it.

Heat lost by silver = mass × specific heat × fall in temperature
$= 200 \times s \times (100 - 21.4)$

Heat gained by water = mass × specific heat × rise in temperature
$= 250 \times 1 \ (21.4 - 18) = 850$ cal.

Heat gained by calorimeter $= 100 \times 0.09 \times (21.4 - 18) = 30.6$ cal.

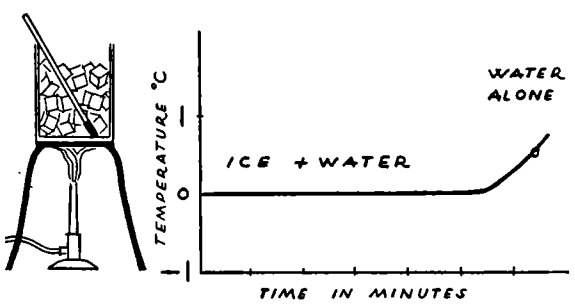
Assuming no heat losses or gains from the outside:

Heat lost by silver = heat gained by calorimeter + water

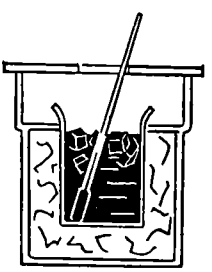
$200 \times s \times 78.6 = 30.6 + 850 = 880.6$ cal.

Specific heat of silver $s = \dfrac{880.6}{200 \times 78.6} = .056$

THERE ARE cases where adding heat to a substance does not change its temperature. In such cases the substance changes its *state*, i.e., goes from a solid to a liquid, or from a liquid to a gas. If you supply heat to a mixture of ice and water at 0° C., the temperature remains at 0° C. until all of the ice is melted. The heat which has been added to melt the ice does not show itself by a rise in temperature and is called LATENT HEAT. The heat energy has been used in changing the state of the substance from solid to liquid. **The number of calories required to change 1 gm. of a substance from the solid to the liquid state without any rise in temperature is called the latent heat of fusion.**



EXAMPLE. Let us use the calorimeter to determine the latent heat of fusion of ice. A copper calorimeter whose specific heat is 0.09 and whose mass is 100 gm. contains 250 gm. of water at a temperature of 25° C. When 30 gm. of carefully dried ice is placed in the calorimeter, the temperature falls from 25° to 14.1° C.



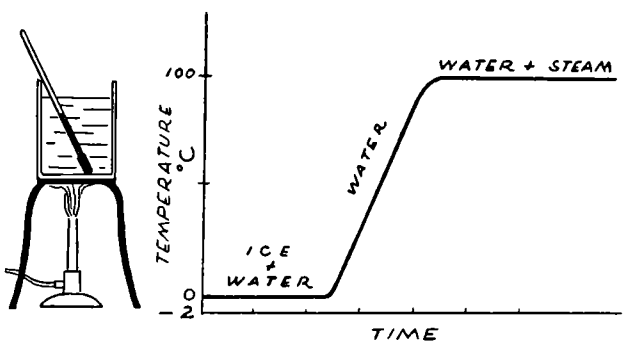
The ice has received not only enough heat to melt it but also enough to raise the temperature of the resulting water from 0° to 14.1° C. This heat has been taken from the calorimeter and water in it, causing their temperature to drop from 25 to 14.1° C.

Heat lost by calorimeter = mass × specific heat × loss in temperature
$= 100 \times 0.09 \times (25 - 14.1)$
$= 100 \times 0.09 \times 10.9 = 98$ cal.

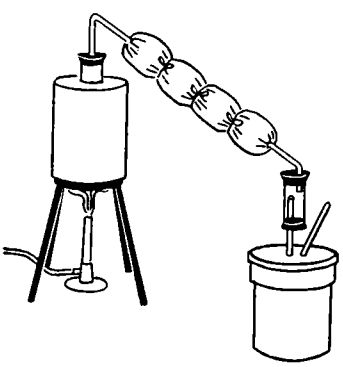
Heat lost by water	$=250\times1\times(25-14.1)$
	$=250\times1\times10.9=2,725$ cal.
Heat required to change ice at 0° C. to water at 0° C.	= mass of ice × latent heat of ice
	$=30L$
Heat required to change water at 0° C. to water at 14.1° C.	= mass × specific heat × rise in temperature
	$=30\times1\times14.1=423$
Heat gained by ice	= heat lost by water + calorimeter
$30L+423$	$=98+2,725$
$30L$	$=2,823-423$
L	$=\frac{2,400}{30}=80$ cal. per gm.

(The accepted value for the latent heat of fusion of ice is 79.63 cal. per gm.)

IF WE continue to heat the water after the ice has melted, we shall find that the temperature continues to rise until it reaches 100° C., when it will again remain constant for a time. The water begins to boil and steam is produced. For every gram of water at 100° C. changed to steam at 100° C., 540 cal. are required.‡



The number of calories required to vaporize 1 gm. of a liquid when its temperature remains constant is called the latent heat of evaporation.



The apparatus shown in the diagram can be used to determine the latent heat of evaporation of water. Water is heated in a boiler and the steam coming off passed into a calorimeter which contains water.** This steam warms up the calorimeter and water. After the temperature has risen a sufficient amount, the steam is shut off.

‡ The actual temperature at which water boils depends on the pressure. At 76 cm. of mercury its boiling point is 100° C.

** Some of the steam will condense on its way to the calorimeter, so to prevent this water being carried to the calorimeter a "steam trap" is used.

EXAMPLE. A copper calorimeter whose specific heat is 0.09 and whose mass is 100 gm. contains 250 gm. of water at 15° C. Dry steam is then passed into it until the temperature reaches 25° C. The calorimeter with the water and the condensed steam has a mass of 354.2 gm. so that the mass of steam which condensed is 4.2 gm. Find the latent heat of evaporation of water.

Heat gained by calorimeter $= \text{mass} \times \text{specific heat} \times \text{rise in temperature}$
$= 100 \times 0.09 \times (25 - 15)$
$= 100 \times 0.09 \times 10 = 90$ cal.

Heat gained by water in calorimeter $= 250 \times 1 \times (25 - 15) = 250 \times 1 \times 10$
$= 2{,}500$ cal.

Heat lost by steam in condensing from steam at 100° C. to water at 100° C. $= \text{mass of steam condensed} \times \text{latent heat}$
$= 4.2 \times L$

Heat lost by this water at 100° C. in cooling to 25° C. $= \text{mass} \times \text{specific heat} \times \text{fall in temperature}$
$= 4.2 \times 1 \times (100 - 25) = 4.2 \times 1 \times 75$
$= 315$ cal.

Heat lost by steam $=$ heat gained by water + calorimeter

$4.2L + 315 = 2{,}500 + 90$

$42L = 2{,}275$

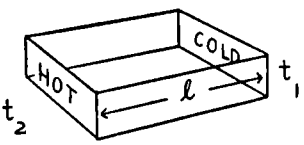
$L = \frac{2{,}275}{4.2} = 540$ cal. per gm.

(The accepted value for the heat of evaporization of water at 100° C., i.e., the latent heat of steam, is 538.7 cal. per gm.)

IN THESE experiments on calorimetry we have assumed that all the heat lost by the hot object is gained by the cold object, none escaping to or entering from the outside. While this is almost impossible to realize in practice, it is possible to minimize this effect. Let us consider the various ways by which a hot body can lose heat. It may do so in three ways: (*a*) by CONDUCTION; (*b*) by CONVECTION; (*c*) by RADIATION.

CONDUCTION MAY be illustrated by placing one end of a poker in a hot fire. The end in the fire becomes red hot and the heat is conducted from the hot end along the poker to the cold handle by molecular agitation. For conduction to take place along the poker, one end must be hotter than the other.

Suppose we have a solid rectangular block one of whose ends is hotter than the other. Let A sq.cm. be the area of cross-section of the block and l its length. The hot end has a temperature $t_2°$ C. while the cold end is at $t_1°$ C. Heat is conducted through the block from the hot to the cold end, the amount being proportional to



1. The area of cross-section, A sq.cm.
2. The time in sec., T.
3. The temperature gradient, $\frac{(t_2-t_1)}{l}$.

If Q is the number of calories conducted through the block in T sec., Q is proportional to $A\times T\times\frac{(t_2-t_1)}{l}$. The proportionality may be removed by introducing a constant:

$$Q=K\times A\times T\times\frac{(t_2-t_1)}{l}$$

K will be a constant for any particular substance such as silver, copper, etc., but will be different for the different substances. The constant K is a measure of the ability of a substance to conduct heat and is known as the THERMAL CONDUCTIVITY of the substance. Thermal conductivity of various substances:

Substance	K, Thermal Conductivity	Substance	K, Thermal Conductivity
Aluminum	0.50	Asbestos	0.0004
Copper	.92	Window glass	.0025
Iron	.16	Ice	0.004
Silver	0.97		

EXAMPLE. Let us find how much heat is conducted per second through a pane of glass 100×60 sq.cm. in area and 2.5 mm. thick, on a cold winter's night when the outside temperature is $-20°$ C. and the room temperature is 20° C. The thermal conductivity K of window glass is 0.0025.

$$Q=\frac{KA(t_2-t_1)T}{l}\qquad K=0.0025\ ;\quad A=100\times60\text{ sq.cm.}$$
$$l=2.5\text{ mm.}=0.25\text{ cm.}\ ;\quad T=1\text{ sec.}$$
$$Q=\frac{0.0025\times100\times60\times40\times1}{0.25}=\frac{2.5\times6\times4\times10^2}{2.5}=2400\text{ cal.}$$

If 2,400 cal. of heat are escaping through the window pane every second, you can see why the radiators need to be on full steam.

CONVECTION is the process by which heat is conveyed throughout a fluid by an actual movement of the hot fluid. Convection takes place in a gas or a liquid due to the fact that the heated portion becomes less dense than the rest and hence rises. Hot-water radiators usually work on this principle. If we wish to prevent heat losses due to convection we must stop the circulation of the heated portions. That is why we place loose cotton between the calorimeter and the outside vessel. Air is a bad conductor of heat, so a woolen blanket with its many air pockets forms a warm covering.

RADIATION, unlike conduction or convection, takes place without the intervention of a material substance. Like light, it travels with the enormous velocity of light, 3×10^{10} cm. per sec. The type of radiation or "the color of the light" depends on the temperature of the object which is radiating. If the object is cool, this radiation is invisible; but as the object is heated, it appears first as a dull red, then yellowish, etc. This we shall discuss later (p. 185). In the case of objects at the same temperature, a black one radiates more heat than does a bright one.

A thermos bottle, or Dewar flask, gives a good illustration of how heat losses or gains due to convection, conduction, and radiation may be made as small as possible. Such a flask was first made by Dewar as a container for liquid air. Two vessels, usually of glass, are placed one within the other and their tops hermetically sealed together. The space between the two flasks is evacuated to a very high vacuum. Thus convection and conduction of heat cannot take place to any great extent. Radiation is cut down by silvering the inside walls.

PROBLEMS

1. *a*) Find how much heat is conducted per hour through each sq.cm. of a sheet of ice on a pond 10 cm. thick when the water below is at 0° C. and the air above is at −20° C.? (The thermal conductivity of ice is 0.005 c.g.s. units.)
   b) How much would the thickness of the ice be increased per hour assuming that the rate of flow of heat remains constant? (Density of ice = 0.9 gm. per cc.)

2. The lady is disconsolately looking at the streaks on her ceiling. She is wondering how they got there. Can you help her? Suppose the ceiling is

built of a series of wooden laths about 3 cm. wide with about half that distance between them. The laths are then completely plastered over. (The thermal conductivity of plaster is greater than that of wood.)

a) Will more heat be conducted through the plaster or through the plaster and wood?

b) Will a place where there is plaster alone be warmer than, cooler than, or the same temperature as a place where the plaster is backed up by the lath? (The ceiling is at a higher temperature than the attic above.)

c) Would you expect dust to settle on the warmer or cooler places?

d) Does the dust settle on the ceiling along the laths or between the laths?

3. On a summer's day, when the water in your drinking glass is at 30° C., you cool it down by putting in two cubes of ice each 4 cm. on a side.
   a) If the density of ice is 0.9 gm. per cc., how many grams of ice do you put in the glass?
   b) If your glass contains 500 cc. of water, find the temperature of the water in it after all the ice is melted, assuming that water alone is cooled.

4. While playing a game of tennis you lose 500 cc. of water by evaporation. (Latent heat of evaporation of water is 540 cal. per gm.)
   a) How many calories of heat have you lost?
   b) If all of this heat was due to playing tennis, how many ergs of work did you do?

5. On a winter picnic you have to use ice to make the coffee. Find how many calories of heat you must supply to 1,000 gm. of ice at −20° C. to bring it to boiling water at 100° C. (The specific heat of ice is 0.5.)

6. An old-fashioned way of warming ale was to put a red-hot poker into it. Suppose the end of an iron poker has a mass of 100 gm. and has been heated to a temperature of 500° C. It is placed in 500 gm. of ale. The temperature of the ale rises from 15° to 30° C. If the specific heat of iron is 0.14, what is the specific heat of the ale?

7. When a lead bullet whose mass is 5 gm. is shot into a block of wood, enough heat is produced just to melt the bullet. The temperature of the

bullet is 27° C. when shot into the block. The melting point of lead is 327° C. (The specific heat of lead is 0.03 and its latent heat is 5.9 cal. per gm.)

a) How much heat was required to melt the bullet?

b) If this energy all came from the *KE* of the bullet, find its *KE* when it entered the wood.

8. The specific heat of olive oil is determined by the method of mixtures. A copper calorimeter whose mass is 100 gm. and whose specific heat is 0.09 is filled with 141 gm. of water at 10° C. After 50 gm. of olive oil at a temperature of 75° C. is added to the calorimeter, the temperature of the mixture is found to be 20° C. Find the specific heat of the olive oil.

9. A steam radiator has a total surface area of 1,000 sq.cm.

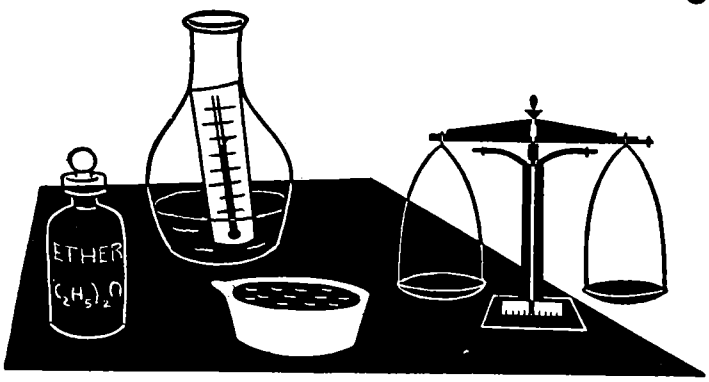
a) How much heat per second is conducted through the iron radiator into a room at a temperature of 20° C. if the thickness of the iron of which the radiator is made is 3 mm.? (Thermal conductivity of iron is 0.15 c.g.s. units.)

b) If this heats a room 4 m. wide, 6 m. long, and 3 m. high, find how many grams of air there are to be heated. (Density of air in room is 0.0012 gm. per cc.)

c) Find how much the temperature of the air in the room is increased per second. (Specific heat of air = 0.24.)

d) Which of the three methods of transferring heat—conduction, convection, or radiation—is the principal one by which heat from a steam radiator is transferred to all parts of a room?

CHAPTER 9



THE MOLECULAR AND KINETIC THEORY OF MATTER

SO FAR we have carefully avoided any attempt to discuss the structure of substances. We have been interested in some of their physical properties, how they behave under the action of forces, etc. Speculations were made by the Greeks some twenty-five hundred years ago as to what they would come to if they were able to break a substance up into exceedingly fine portions. Would they eventually reach some ultimate unit characteristic of the substance as a whole? They believed that they would, and to this ultimate entity they gave the word *atom*, meaning "indivisible." Such speculations without experimental evidence lead nowhere, so the atomic theory cannot be said to have had any scientific foundation until we come to the eighteenth century and the brilliant work of the chemists. From the careful work of Dalton, Avogadro, and many others there finally evolved the idea of the *atom* and the *molecule*.

AN ATOM IS THE SMALLEST ELECTRICALLY NEUTRAL PARTICLE OF AN ELEMENT THAT CAN TAKE PART IN A CHEMICAL CHANGE. There are ninety-two different elements, e.g., copper, oxygen, sodium, etc.

A MOLECULE IS THE SMALLEST PARTICLE OF AN ELEMENT OR A COMPOUND WHICH CAN NORMALLY LEAD A SEPARATE EXISTENCE. The hydrogen molecule, H_2, consists of two hydrogen atoms and for this reason is called a diatomic molecule. There are some molecules which contain only one atom. The inert gases, helium (He), argon (A), neon (Ne), etc., are all monatomic. In the case of such gases the atom and the molecule are the same. The water molecule is built up of hydrogen and oxygen atoms, two atoms of hydrogen and one atom of oxygen combining to form one molecule of water, H_2O. The

molecules of some compounds are made up of a very large number of atoms. The starch molecule, e.g., contains at least ten thousand atoms.

The clear distinction between atoms and molecules in the mind of an Italian, Amedeo Avogadro, enabled him in 1811 to advance an important hypothesis which now bears his name. **Avogadro's hypothesis: Equal volumes of gases at the same temperature and pressure contain the same number of molecules.** Molecules of different elements have different masses and hence different weights. The weight of any individual molecule is so small that rather than use the weight of a single molecule its weight in terms of some standard molecule is used. The unit chosen is $\frac{1}{32}$ of the molecular weight of oxygen, this being taken as 32. Now 32 gm. of oxygen at STP occupies a volume of 22.4 liters (1 liter $= 10^3$ cc.) and contains 6.06×10^{23} molecules. This number, 6.06×10^{23}, is called Avogadro's number.

By Avogadro's hypothesis 22.4 liters of any gas at STP will contain 6.06×10^{23} molecules. Now 22.4 liters of hydrogen at STP weighs 2 gm. Thus the molecular weight of hydrogen is 2. The oxygen molecule is therefore 16 times as heavy as the hydrogen molecule.† THE WEIGHT IN GRAMS OF 22.4 LITERS OF ANY GAS AT STP IS NUMERICALLY EQUAL TO THE MOLECULAR WEIGHT OF THE GAS. With this information and a table of molecular weights we can very simply find the densities of gases.

THE DENSITY of oxygen at STP: The weight of 22.4 liters of oxygen at STP is 32.00 gm., numerically equal to the molecular weight of oxygen.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{32.00}{22.4 \times 10^3} = 1.43 \times 10^{-3} = .00143 \text{ gm. per cc.}$$

$\therefore$ The density of oxygen at 0° C. and 76 cm. of mercury is .00143 gm. per cc.

WE MAY find the relative density of a gas with respect to that of hydrogen if we know the molecular weights of each gas. Now 2.016 gm. of hydrogen gas, 4.00 gm. of helium gas, 32.00 gm. of oxygen gas each have a volume of 22.4 liters at STP. Each of these masses contains the same number of

Gas*	Hydrogen	Helium	Oxygen	Nitrogen
Atomic weight	1.008	4.00	16.00	14.00
Molecular weight	2.016	4.00	32.00	28.00

* Notice that hydrogen and oxygen are diatomic molecules while helium is monatomic.

† The foregoing figures are only approximate.

molecules, 6.06×10^{23}. The density of a gas is its mass per unit volume. Oxygen is $\frac{32.00}{2.016}$ times as heavy as hydrogen. Therefore the density of oxygen is 15.9 times as great as that of hydrogen. If we may neglect any departure from the gas laws, these relative densities will be the same at any temperature and pressure.

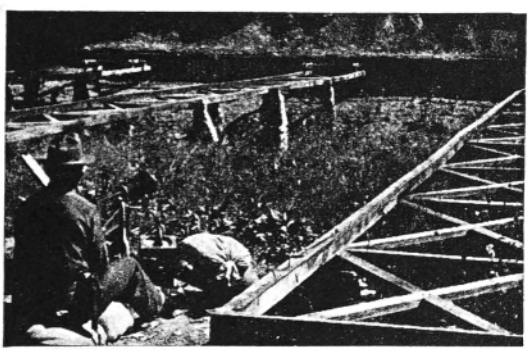
KINETIC THEORY OF GASES

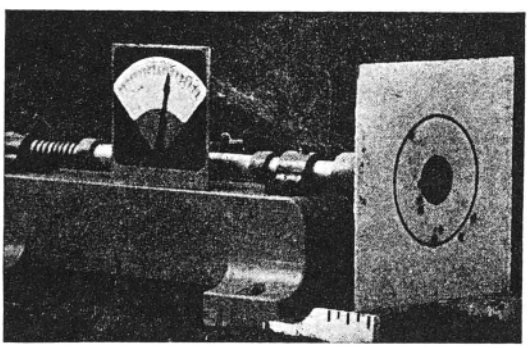
The number of molecules in 1 cc. of a gas at STP is

$$\frac{6.06\times10^{23}}{22.4\times10^{3}}=2.70\times10^{19}\text{ molecules}$$

A cubic centimeter is a relatively small space, yet in this space there are twenty-seven million, million, million molecules. Even now there is plenty of empty space between the molecules. These molecules of a gas are in constant motion, banging on the walls of the containing vessel and exerting a gas pressure.

EXAMPLE. The problem of calculating the pressure which the molecules exert on the walls of the vessel is similar to that of calculating the pressure exerted by bullets fired at a target





From the Educational Sound Picture, "The Molecular Theory of Matter," by H. I. Schlesinger and H. B. Lemon, produced by Erpi Picture Consultants, Inc., and the University of Chicago Press (copyright University of Chicago)

from a machine gun. Every time a bullet hits the target, the target receives momentum. Let us find the pressure exerted on a steel target 100 sq.cm. in area when 200 machine-gun bullets, each 5 gm. in mass, strike every second. The velocity of each of the bullets is 20,000 cm. per sec. We shall assume that the collisions between the bullets and the target are elastic.

The momentum of each bullet as it strikes the target is

$$mv=5\times20{,}000=10^{5}\text{ gm. cm. per sec.}$$

The bullet rebounds with the same velocity with which it struck, so that its momentum on rebound is -10^{5} gm. cm. per sec.

$$\text{Change in momentum of bullet}=10^{5}-(-10^{5})=10^{5}+10^{5}=2\times10^{5}\text{ gm. cm. per sec.}$$

From the Law of Conservation of Momentum, the change in momentum of the target must be 2×10^{5} gm. cm. per sec. for each bullet striking it. But 200 bullets arrive every second, so

that the total momentum change of the target every second is $200 \times 2 \times 10^5$, i.e., 4×10^7 gm. cm. per sec. This change in momentum every second is also the force acting on the target $\left(F = \frac{mv_c}{t}\right)$. Therefore the force on the target is 4×10^7 dynes.

$$\text{Pressure exerted by bullets on target} = \frac{\text{force}}{\text{area}} = \frac{4 \times 10^7}{100} = 4 \times 10^5 \text{ dynes per sq.cm.}$$

According to the kinetic theory, the molecules of a gas are small hard particles which move rapidly and make elastic collisions whenever they strike the walls of the vessel or each other. Suppose N molecules of a gas are inclosed in a vessel of volume V and that the molecules have a mass* m and a velocity v. Then the pressure exerted by the molecules† is

$$p = \frac{1}{3} \times \frac{Nmv^2}{V}$$

(The proof of this equation may be found in Lemon, *From Galileo to the Nuclear Age*, p. 158.)

THE KINETIC theory of gases may be used to interpret Boyle's and Charles's laws. Boyle's Law states that the product pV is a constant when the temperature is constant. From kinetic theory.

$$\begin{aligned} pV &= \tfrac{1}{3}Nmv^2; && Nm = \text{total mass of gas} \\ &= \tfrac{2}{3} \times \tfrac{1}{2}Nmv^2; && \tfrac{1}{2}Nmv^2 = \text{total } KE \text{ of the molecules} \\ &= \tfrac{2}{3} \text{ total } KE \text{ of the molecules} \end{aligned}$$

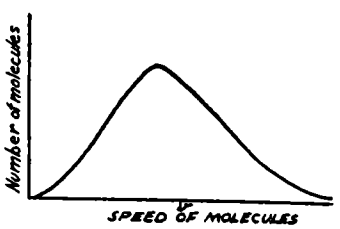
We are now going to assume that the total KE of the molecules is a measure of the absolute temperature T of the gas. Hence

$$\frac{pV}{T} = \text{constant}$$

But this is the general gas law obtained by combining Boyle's and Charles's laws.

* Recall that mass is proportional to weight: $w = mg$. So the fact that we are now using the mass of molecules whereas we previously used their weight need cause us no worry.

† The assumption that all the molecules have the same velocity even when the gas is at uniform temperature is not justified. Maxwell has deduced a distribution law for the velocities of the molecules. The ordinates of the curve give the number of molecules having a particular velocity. If we interpret the v^2 in the foregoing equation as an average of the squares of the velocities of all of the molecules, we shall be much nearer the picture.



EXAMPLE. We shall now find the velocity of the oxygen molecules at 0° C. and 76 cm. of mercury. The pressure p of the gas is $\frac{1}{3}\frac{Nmv^2}{V}$, but Nm is the total mass of gas in a volume V, so that $\frac{Nm}{V}$ is the density, ρ, of the gas. Therefore $p=\frac{1}{3}\rho v^2$.

$$v^2=\frac{3p}{\rho}\ ; \qquad p=76.0\times 980\times 13.6=1.01\times 10^6 \text{ dynes per sq.cm.}$$

$$\rho=1.43\times 10^{-3} \text{ gm. per cc. for oxygen at STP.}$$

$$v^2=\frac{3\times 1.01\times 10^6}{1.43\times 10^{-3}}=\frac{3.03\times 10^9}{1.43}=2.12\times 10^9 \text{ (cm. per sec.)}^2$$

The mean velocity of the molecules is obtained by taking the square root of v^2. This root mean square velocity we shall call v_m. So that

$$v_m=\sqrt{2.12\times 10^9}=4.60\times 10^4=46{,}000 \text{ cm. per sec.}$$

AVOGADRO'S HYPOTHESIS that equal volumes of gases at the same temperature and pressure contain the same number of molecules can be predicted from the kinetic theory of gases. Let us use the equation $pV=\frac{1}{3}Nmv^2$ and take two gases at the same volume, temperature and pressure.

$$\text{For gas 1: } p_1V_1=\tfrac{1}{3}N_1m_1v_1^2$$

$$\text{For gas 2: } p_2V_2=\tfrac{1}{3}N_2m_2v_2^2$$

Since the volumes and pressures are equal, $V_1=V_2$ and $p_1=p_2$.

$$\therefore \quad p_1V_1=p_2V_2$$

or

$$\tfrac{2}{3}\times\tfrac{1}{2}(N_1m_1v_1^2)=\tfrac{2}{3}\times\tfrac{1}{2}(N_2m_2v_2^2)$$

The temperatures are the same; therefore the mean KE of the molecules of the two gases are the same.

$$\tfrac{1}{2}m_1v_1^2=\tfrac{1}{2}m_2v_2^2$$

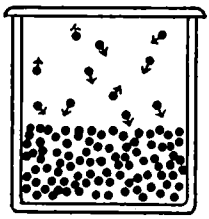
$$\therefore \quad \tfrac{2}{3}N_1=\tfrac{2}{3}N_2 \qquad \text{or} \qquad N_1=N_2$$

The two gases contain the same number of molecules. Thus from our kinetic theory of gases we have deduced Avogadro's hypothesis.

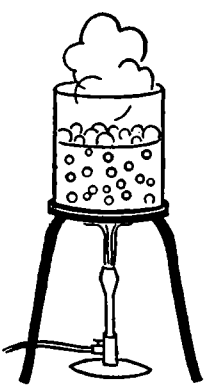
VAPOR PRESSURE: SATURATED AND UNSATURATED VAPORS

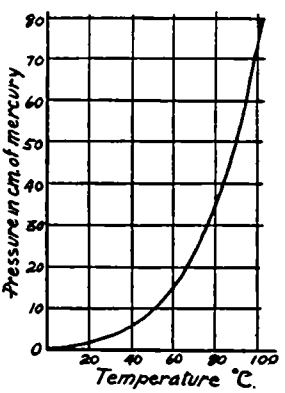
In a liquid which is uniform in temperature, some molecules may move faster than others. There is a distribution of velocity about some mean value as in the case of a gas. In this case the faster-moving ones at the surface may escape at the surface of liquid. Then the average velocity of the remaining molecules is lower. The liquid is cooled by evaporation.

Consider a liquid in a closed vessel from which all the air has been removed. Some of the faster-moving molecules escape into the space above, but, having escaped, some of them will return into the liquid. The molecules flying about in the space above the liquid exert a pressure which in this case we call VAPOR PRESSURE. The greater the number of the molecules in the space above, the greater the number returning to the liquid. Eventually a balance is reached where the number of molecules leaving the liquid per second will be the same as the number returning per second. When the number of molecules above the liquid is at the maximum for the particular temperature, the vapor is saturated and is exerting its *maximum or saturation vapor pressure.* The number of molecules in the space above the liquid and hence the saturation vapor pressure will depend only on the temperature. If there were any air in the space above, the same saturation vapor pressure would be reached, but it would take a longer time for this pressure to be established than when the space was empty.



The graph shows the maximum or saturation vapor pressure curve for water at different temperatures. The maximum vapor pressure for water at 100° C. is 76 cm. of mercury. The boiling point of water at this pressure is 100° C. This is no accident. Let us consider what we mean by the boiling of a liquid. When water boils, bubbles of water vapor are formed in the liquid. These bubbles have a pressure inside them equal to the maximum vapor pressure of water at the temperature of the water. The pressure tending to cause them to collapse is the atmospheric pressure plus an almost negligible pressure due to the water itself. The bubbles can form only when the maximum vapor pressure of the liquid is equal to the pressure on the surface of the liquid. *Thus the maximum vapor pressure curve is also the boiling-point curve.* At an altitude of about a mile, atmospheric pressure is about 62 cm., so that water would boil at about 94°.5 C.

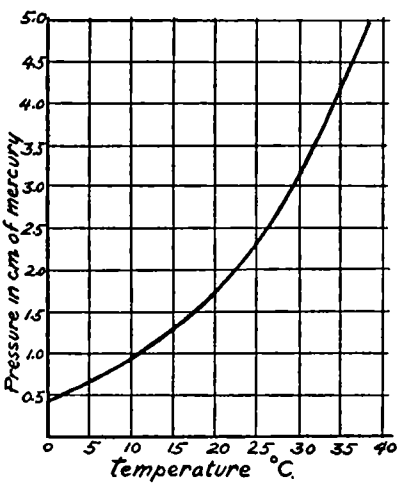




Maximum vapor pressure for water at various temperatures

RELATIVE HUMIDITY. You have probably all seen the dew which forms on the outside of your ice-cold water glass. When you read that the relative humidity on that day is 85 per cent, do you know what it means?

It means that the air contains 85 per cent or $\frac{85}{100}$ as much water vapor as it possibly can, i.e., it is 0.85 saturated with water vapor. How can this relative humidity be determined? A simple method is to find the temperature of the warmest water in your glass which will allow dew to be deposited. When dew is formed it means that the air is saturated with water vapor. If we know the temperature at which dew is formed, we can then determine from the vapor-pressure graph the maximum vapor pressure at this temperature. This will be the vapor pressure of the water vapor in the air.



Maximum vapor pressure for water from 0° to 40° C.

Suppose on a day when the air temperature is 35° C. (95° F.) dew forms on the glass when its temperature is 30° C.

$$\left.\begin{matrix}\text{Vapor pressure of water}\\ \text{vapor in the air}\end{matrix}\right\} = \left.\begin{matrix}\text{maximum vapor pres-}\\ \text{sure at 30° C.}\end{matrix}\right\} = 3.15 \text{ cm. of mercury}$$

If the air were saturated with water vapor, it would exert a vapor pressure equal to the maximum vapor pressure of water at 35° C., which is 4.22 cm. of mercury. Now

$$\text{Relative humidity} = \frac{\text{maximum vapor pressure at dew point}}{\text{maximum vapor pressure at temperature of air}}$$

$$= \frac{3.15}{4.22} = .74 = 74 \text{ per cent}$$

This is, of course, a very rough method for determining relative humidity.

PROBLEMS

1. From the table of molecular and atomic weights on page 100 of the text find how many times helium gas is heavier than hydrogen gas.
2. The dirigible "Macon" had a gas volume of 18×10^{10} cc. (6,500,000 cu.ft.) and was filled with helium gas at atmospheric pressure.
   a) What weight could the dirigible lift if filled with helium at STP? (Density of air = 0.00129 gm. per cc. at STP.)
   b) What would be the lifting power of the dirigible if filled with hydrogen at STP?
   c) Why is helium used in preference to hydrogen?

3. A Coolidge X-ray tube, whose volume is 1000 cc., is evacuated to the highest possible vacuum (approximately 10^{-9} cm. of mercury). Find the total number of molecules still remaining in the X-ray tube.

4. The gas, carbon dioxide (CO_2), has a molecular weight of 44.
   a) How many molecules of CO_2 are there in 44 gm. of the gas at STP?
   b) What volume does 44 gm. of CO_2 occupy at STP?
   c) What volume does 44 gm. of CO_2 occupy at 20° C. and 70 cm. of mercury?

5. Dalton showed experimentally that, when several gases which do not react chemically with each other are contained in a closed vessel, the total pressure is the sum of the pressures which each gas would exert separately. Show from the kinetic theory that this is true for a mixture of two gases. (Call the gases 1 and 2.) Then the pressure exerted by gas 1 will be $p_1 = \frac{1}{3}n_1m_1v_1^2$, and for gas 2 will be $p_1 = \frac{1}{3}n_2m_2v_2^2$. The temperature of the two gases is the same.

6. *a*) Do hydrogen molecules at STP move faster than oxygen molecules at STP?
   b) Do carbon dioxide molecules at STP move faster than oxygen molecules at STP?

7. In the text on page 103, we calculated v^2, the average of the squares of the velocities of the oxygen molecules and found it to be 2.12×10^9 (cm. per sec.)^2. Find v^2 for the hydrogen and nitrogen molecule, using the table of molecular weights on page 100.

8. Is the space above the mercury in a mercury barometer really a vacuum or does it contain mercury molecules? If it does contain mercury molecules, does the number of them, and hence the pressure which they exert, vary with temperature?

9. On a mountain does water boil at a temperature higher or lower than 100° C.?

10. From the vapor-pressure graph on page 104, find the boiling point of water on top of Mount Everest, which is 29,500 ft. high and has an average barometric pressure of 24 cm. of mercury.

11. Mount Whitney in California is about half as high as Mount Everest. The average barometric pressure at its top is 43 cm. of mercury. Find the boiling point of water on top of this mountain.

12. On a hot summer's day when the temperature is 35° C., dew forms on the outside of your water glass when the temperature of the water is 25° C.
    a) What is the dew point on this day?
    b) What is the relative humidity?

13. Would it be any better to use a silver vessel instead of an ordinary glass when determining the dew point? Why?

14. Why is a dry, hot day pleasanter than an equally hot day which is humid?

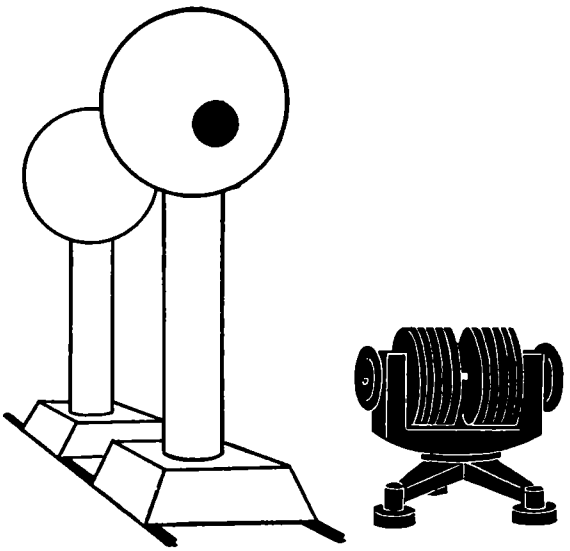
15. Why does a breeze help to make a hot humid day so much pleasanter?

16. A closed vessel from which ALL of the air has been removed is partially filled with water. Could the water in the vessel be made to boil?

17. A closed vessel containing air is partially filled with water. Could the water in the vessel be made to boil?

18. From the kinetic theory point of view why does dust move from the warmer to the cooler places in a room?

CHAPTER 10



ELECTRIC CHARGES AND MAGNETIC POLES

IT IS relatively easy to produce electric charges. Take your fountain pen and rub it vigorously with your handkerchief. You will find that it attracts small pieces of paper. The fountain pen and handkerchief have been electrified. It is possible to electrify almost anything by rubbing it with a suitable substance. There are two kinds of electric charges, called, for convenience, positive (+) and negative (−). The fountain pen acquires a negative charge while the handkerchief acquires a positive charge. On the contrary, a glass rod rubbed with silk becomes positively charged and the silk negatively. A few simple experiments soon establish the fact that a positively charged object repels another positively charged object but it attracts a negatively charged object. These results may be summarized: UNLIKE CHARGES ATTRACT AND LIKE CHARGES REPEL EACH OTHER.

Coulomb showed experimentally that, if two like electric charges of magnitude Q_1 and Q_2 are placed a distance r cm. apart in air, they repel each other with a force of $\frac{Q_1 \times Q_2}{r^2}$ dynes. This law of force leads to a definition of the unit of electrostatic charge (1 e.s.u. of charge). ONE E.S.U. OF CHARGE IS THAT CHARGE WHICH WHEN PLACED 1 CM. IN AIR FROM AN EQUAL AND SIMILAR CHARGE REPELS IT WITH A FORCE OF 1 DYNE.*

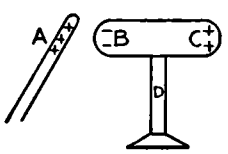
* It can be shown that the force between two charges depends not only on the magnitude of the charges Q_1 and Q_2 and their distance apart r, but also on the medium (air, water, paraffin, etc.) between the two charges. We shall then re-write the foregoing equation:

$$F = \frac{Q_1 \times Q_2}{Kr^2}$$

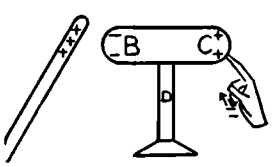
[Footnote continued on opposite page.]

YOU HAVE probably heard of the fundamental unit of negative charge, the ELECTRON. No charge of electricity has ever been obtained smaller than the electron. All electric charges are multiples of this charge. The negative charge on the electron is 4.77×10^{-10} e.s.u. We should have to have almost 2,100,000,000 electrons to make a total charge equal to the electrostatic unit of charge. It is the free electrons which carry the electric current in a metallic wire. Substances which conduct an electric current and hence have free electrons are called *conductors*. There are also substances, such as glass, paraffin, rubber, etc., in which the electrons are tightly bound to the atom and cannot move. These substances do not conduct electricity and are called *insulators*. A good insulator has a high electrical resistance and a good conductor a low electrical resistance. There is no substance which is a perfect insulator, but there are all gradations between good insulators such as glass and good conductors such as silver or copper.

The fact that electrons flow readily in a conductor allows us to charge a conductor by bringing another charged object close to it. This is called charging by *induction*. Suppose a glass rod A, positively charged by rubbing with silk, is brought close to a metal object BC supported on an insulator D.



The presence of the charged rod at A causes electrons to flow in BC toward A. This means that the end B will have an excess of electrons, while C has a deficiency. Thus B has an *induced* negative charge and C an *induced* positive charge due to the presence of the positively charged rod A. If A is removed, there is nothing to keep the charges at B and C separated. The electrons flow back and the charges are neutralized. However if, while A is present, the end C is touched with a wire or a finger (the human body is a good enough conductor for this purpose), electrons from the earth will be attracted by the positive charge at C and will neutralize this charge. If now you break your finger connection before the charged rod is removed, the excess negative charge is trapped on BC and will spread over BC when A is



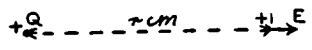
K is called the *dielectric constant* of the medium. The values of K for different substances are determined experimentally. A few are given in the table below:

Substance	Air or Vacuum	Water	Paraffin	Glass
Dielectric constant K.....	1	81	2	7

taken away. Can you say what would happen if A were removed while the finger is still in contact?§

The amount of charge induced on BC will depend on the charge on A and the distance of A from BC. It depends on the *electric intensity*, i.e., *electric field strength*, produced by A at BC. BY ELECTRIC INTENSITY OR ELECTRIC FIELD STRENGTH AT ANY POINT we mean THE FORCE WHICH WOULD BE EXPERIENCED BY A UNIT POSITIVE CHARGE PLACED AT THAT POINT.

EXAMPLE. What is the strength of the electric field E at a distance r cm. from a positive charge Q? We must find the force exerted on a unit positive charge placed a distance of r cm. from Q. The force on the charges Q_1, Q_2 placed r cm. apart in air is $F=\frac{Q_1Q_2}{r^2}$. In the present case $Q_1=Q$ and $Q_2=+1$.



$$\therefore \quad F=\frac{Q\times 1}{r^2} \qquad \text{or} \qquad \frac{Q}{r^2} \text{ dynes}$$

The electric field E at a distance of r cm. from a charge Q is $\frac{Q}{r^2}$ e.s.u. of field strength.† The field is directed away from Q since the force between $+Q$ and the unit positive charge is one of repulsion.

SINCE THE force on 1 e.s.u. of charge placed in a field of strength E is E dynes, the force on Q units of charge would be QE dynes.

EXAMPLE. Find the electric field strength E at a point 5 cm. from a charge of $+100$ e.s.u. Find the force exerted on a charge of $+8$ e.s.u. situated at this point.

$$\text{Electric field strength: } E=\frac{Q}{r^2}=\frac{100}{25}=4 \text{ e.s.u.}$$

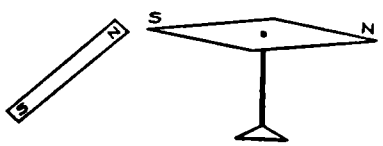
The force on a charge of $+8$ e.s.u. situated at this point is 4×8, i.e., 32 dynes. The force is one of repulsion. The force on a charge of -8 e.s.u. at the point would be 32 dynes, but it would now be one of attraction. Rather than pursue the subject of electric field further, we shall turn our attention to some simple ideas in magnetism.

THERE IS an iron compound, Fe_3O_4, sometimes known as lodestone, which if hung freely by a fine suspension will set itself along an approximate north-south line. This was the earliest magnetic compass. Nowadays a magnetized steel bar or needle carefully supported on a vertical axis is the basis of the mariner's compass. A magnetized bar attracts small pieces of iron. This attraction is most noticeable at two points in the magnet. These are called the NORTH (N.) pole and the SOUTH (S.) pole. If we should cut this magnet so as to try and separate the N. and S. pole, we should find it impossible; each time we cut the magnet we would have two other magnets, each having

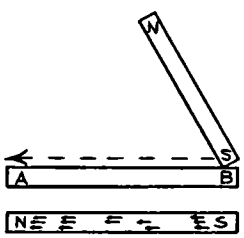
† The unit of electric field strength has no special name. We shall have to call it what it is, i.e., the electrostatic unit of field strength.

a N. and a S. pole. We can never get an isolated N. or S. pole as we can an isolated positive or negative charge. The *N. pole* of a magnet is that pole which points *toward* the geographic north when the magnet is freely suspended.‡

IF YOU take a magnet and a compass needle you can easily show yourself that a N. pole repels a N. pole and attracts a S. pole. LIKE POLES REPEL AND UNLIKE POLES ATTRACT EACH OTHER. The law of force between two poles is similar to that for the force between two electric charges, or two masses. TWO LIKE POLES OF STRENGTHS P_1 AND P_2 A DISTANCE OF r CM. APART IN AIR REPEL ONE ANOTHER WITH A FORCE OF $\frac{P_1 \times P_2}{r^2}$ dynes. From this law we get our definition of unit pole strength. A UNIT MAGNETIC POLE IS THAT POLE WHICH WHEN PLACED 1 CM. IN AIR FROM AN EQUAL AND SIMILAR POLE REPELS IT WITH A FORCE OF 1 DYNE.



ONLY A few substances can be magnetized to a perceptible degree. They are iron, nickel, cobalt, and some alloys. We can easily magnetize an iron bar by repeatedly drawing a magnet over it in the same direction, as shown in the diagram. The end B will have a S. polarity and the end A a N. polarity. It is as though the molecules of a magnetic substance were small magnets and this treatment caused them to orient themselves. This readily accounts for the fact that we can never get an isolated N. or S. pole. However, we shall simplify our discussion if we go ahead with the assumption that we can have isolated poles.**



JUST AS we have an electric field surrounding an electric charge, so we have a magnetic field surrounding a magnet or a magnetic pole. BY MAGNETIC FIELD STRENGTH H AT A POINT WE MEAN THE FORCE IN DYNES ON A UNIT N. POLE PLACED AT THAT POINT. Magnetic field strengths are measured

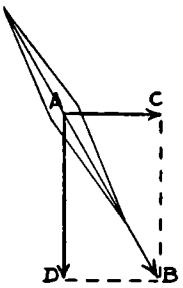
§ Answer to question on the previous page. The excess negative charge on *BC* would have nothing to hold it there and hence would flow back to the earth, leaving *BC* electrically neutral.

‡ Strictly speaking, a freely suspended magnet or compass needle does not set itself along a geographic N.-S. line, but at a small angle to it. This angle is different for different places on the earth's surface. Careful measurements have been made of this angle, so that the geographic N. can be obtained from the direction of the compass needle.

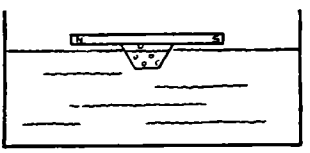
** This assumption in no way invalidates our arguments; the simplification permits us to speak of forces instead of moments of forces.

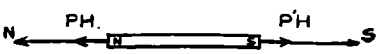
in *gauss*. If a unit pole is placed in a field of *1 gauss*, it is acted on by a force of *1 dyne*. A pole of strength P placed in a field of strength H *gauss* is acted on by a force of $P \times H$ dynes.

The earth itself has a magnetic field, the magnitude and direction of which are different at different places on the earth. The angle which the earth's magnetic field makes with the horizontal is called the *angle of dip*. If the vector AB represents the earth's magnetic field, and AC is horizontal, the angle CAB is the angle of dip. The vector AC represents the horizontal component of the earth's magnetic field and AD the vertical component. Over a small area of surface we may consider the earth's magnetic field as constant.



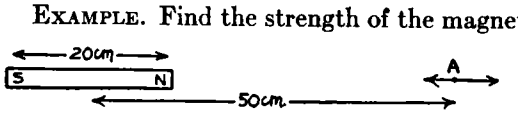
Using this information, we shall now prove that in any magnet the strength of the N. pole is the same as the strength of the S. pole. We shall take a bar magnet and float it by means of a cork on a large bowl of water. We find that the magnet orients itself in the magnetic N.–S. line, but does not move directly toward either the N. or the S. pole. Suppose the strength of the N. pole is P and that of the S. pole is P'. If the strength of the horizontal component of the earth's field is H gauss, the force on the N. pole is $P \times H$ and that on the S. pole $P' \times H$. Since the resultant force on the magnet must be zero, $PH - P'H$ is zero and $P = P'$. Therefore, the strengths of the N. and the S. poles of any magnet are the same.





IF YOU have ever gone on the bridge of a ship you know that the compass is "compensated" for the magnetism of the ship itself by means of suitably placed magnets. The compensation is done by what is known as "swinging the ship"; the ship is placed in some known direction and a reading of the compass taken. Magnets are then placed so as to bring the compass into line. Accurate calculations as to the magnetic field due to the ship's magnetism would be a very complicated problem, so we shall content ourselves with a much simpler one:

EXAMPLE. Find the strength of the magnetic field at a distance of 50 cm. from the center of and on a line with the axis of a magnet. The pole strength of the magnet is 5 units and the distance between the poles is 20 cm. In order to find the strength of the magnetic field at A, we imagine a unit N. pole placed at A. We then find the force on it due to the two poles of the magnet.



Force on unit N. pole at A due to N. pole of magnet:

$$F=\frac{PP'}{r^2}=\frac{5\times1}{(50-10)^2}=\frac{5}{(40)^2}=\frac{5}{1,600}\text{ dynes}$$

Force on unit N. pole at A due to S. pole of magnet:

$$F=\frac{PP'}{r^2}=\frac{5\times1}{(50+10)^2}=\frac{5}{(60)^2}=\frac{5}{3,600}\text{ dynes}$$

These forces are in opposite directions, one being a repulsion and the other an attraction. The resultant force is a repulsion:

Resultant force:

$$F=\frac{5}{1,600}-\frac{5}{3,600}=\frac{5(3,600-1,600)}{1,600\times3,600}=\frac{5\times2,000}{1,600\times3,600}=\frac{1}{16\times36}=\frac{1}{576}$$
$$=0.0017\text{ dynes}$$

The resultant magnetic field is 0.0017 gauss and is directed away from the magnet.

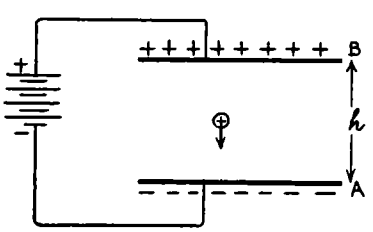
THE SIMILARITY between the law of force for electric charges or magnetic poles and that for two masses,

$$F=\frac{G(m_1\times m_2)}{r^2},$$

has probably already struck you.* We have already introduced the idea of electric field strength E, i.e., the force in dynes acting on unit + charge placed in the electric field. Is there a gravitational field strength due to the earth's attractive force? In other words, what is the force in dynes acting on a unit mass of 1 gm. placed in the earth's gravitational field? This force is 980 dynes, where 980 is the acceleration due to gravity g. The strength, then, of the earth's gravitational field is g. We saw that if we moved a mass m a vertical height h against gravity, i.e., against the earth's gravitational field, we did an amount of work, mgh. We expressed this in another way by saying the potential energy PE of the object of mass m was increased by mgh ergs.

WORK AGAINST AN ELECTRIC FIELD

BY EXACTLY similar reasoning, if we move a charge Q against a uniform electric field of strength E a distance h, we increase the potential energy of the charge by EQh. We can produce a uniform electric field E by connecting a battery to two parallel metallic plates. The upper plate in the diagram has a positive charge on it and the lower plate a negative one. If we place a positive charge Q in the field, it will be acted on by a downward force of EQ; the lower plate with its negative charge attracts Q



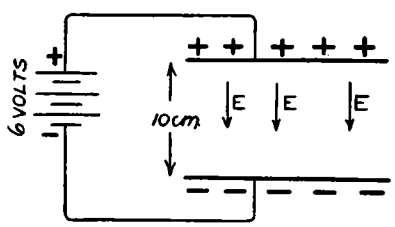
* For our present work we do not need to discuss the magnetic law of force any further.

and the upper plate with its positive charge repels Q. Moving the charge $+Q$ from A to B means we must exert a force of EQ through a distance h and do an amount of work of EQh against the electric field.† If we let go of the charge at B, it would immediately fall toward the lower plate just as the mass m with its PE of mgh would fall if released.

POTENTIAL DIFFERENCE

NOW IN electricity it is usual to talk of the work done when a charge of 1 unit is moved rather than Q units. THE WORK DONE IN MOVING THE UNIT POSITIVE CHARGE FROM A TO B IS Eh, AND THIS IS CALLED THE **potential difference**, PD, BETWEEN A AND B. The unit in which this work is expressed depends on the unit charge moved and the unit of PD used. We are going to use the *practical units*, with some of which you are familiar. The unit of PD is the VOLT, and the unit of charge is the COULOMB. (A coulomb of charge is not the same as the electrostatic unit of charge. It is actually 3×10^9 e.s.u.; but don't remember this.) The relation between these units is such that **if we move 1 coulomb of charge through a PD of 1 volt, 1 joule of work is done. If we move a charge of Q coulombs through a PD of V volts, the work done is $Q\times V$ joules.**

EXAMPLE. Suppose we consider an ordinary automobile battery which by chemical action gives a PD of 6 volts. This is connected to two parallel metallic plates 10 cm. apart. Let us find the work done in moving 1 coulomb of charge from the lower plate to the upper, and also the strength of the electric field between the plates.



The fact that the PD between the plates is 6 volts means that if we move 1 coulomb of positive charge from the lower to the upper plate we must do 6 joules of work. (What energy does 1 coulomb acquire if it falls from the positive plate to the negative plate?)§

The PD between the plates (or the work done in moving a unit positive charge from one plate to the other) is the field strength E multiplied by the distance moved h.

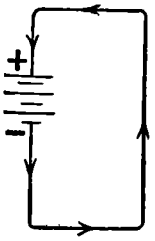
$$PD = E\times h; \qquad PD = 6 \text{ volts}; \qquad h = 10 \text{ cm}.$$

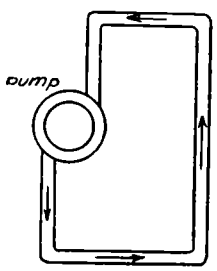
$$\therefore \quad E = \frac{PD}{h} = \frac{6}{10} = 0.6 \text{ volts per cm.}‡$$

† We are neglecting the mass of the charge Q, so no work *against gravity* is done in moving the charge Q from A to B.

‡ An electric field of 0.6 volts per cm. means that if we move 1 coulomb of charge 1 cm. in the field, 0.6 joules, or 0.6×10^7 ergs, of work is done. Our former definition of electric field was that it was the force on unit charge placed in the field. Thus, in this field, there is a force of 0.6×10^7 dynes on 1 coulomb of charge. The work in moving this 1 coulomb 1 cm. in the field is $F\times s$, 0.6×10^7 ergs, or 0.6 joules.

LET US now connect the two terminals of our battery to a wire. The electrons attracted by the positive pole of the battery and repelled by the negative pole will flow in the wire in the direction shown by the arrows. This flowing of the electrons round the circuit is called an *electric current.* In the practical units the electric current I is measured in *amperes.* We say an *ampere* of current is flowing when *1 coulomb of charge flows past any point in the wire in 1 sec.* (This means that actually about 6.3×10^{18} electrons flow past any point in a second.) Contrast this with the rate of flow of water. A pump forces water through the pipes just as the battery forces electrons through the circuit. The rate of flow of water passing every cross-section of the pipe must be the same and could be measured in gallons per second. The number of gallons, or quantity of water, corresponds to the number of coulombs of charge. The rate of flow of water, or the current, is measured in gallons per sec. while the flow of electric current is measured in coulombs per sec., i.e., amperes. Thus, if Q coulombs flow past a point in t sec., Q/t coulombs flow past in 1 sec. This is the current I in amperes:





$$I = \frac{Q}{t}; \qquad \text{Amperes} = \frac{\text{coulombs}}{\text{seconds}}$$

If Q coulombs flow in the circuit in t sec. due to the fact that the battery has a PD of V volts, the work done by the battery is $Q \times V$ joules. These Q coulombs flow round in t sec.

$$\text{Work done per sec.} = \frac{QV}{t} = I \times V \text{ joules per sec.} = I \times V \text{ watts}$$

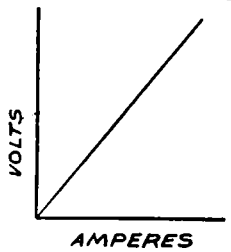
$\text{Work} = Q \times V$ joules; coulombs $\times$ volts = joules

$\text{Power} = I \times V$ watts; amperes $\times$ volts = watts

Voltage may be thought of as an electrical pressure. It is this which forces the electrons to flow in the wire. The number of electrons which flow per sec. past any point, i.e., the current, will depend on the electrical pressure and the electrical resistance of the wire, just as the number of gallons of water flowing through the pipe per sec. depends on the pump pressure and the resistance of the connecting pipes.

§ Answer to question on the previous page: 6 joules of energy.

RELATIONSHIP BETWEEN CURRENT AND VOLTAGE IN A METALLIC CONDUCTOR

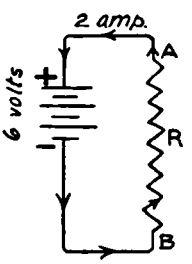


IN 1828 George Simon Ohm found experimentally that the *current flowing in a metallic circuit is proportional to the voltage applied.* If the voltage is doubled, the current is doubled. The relationship between the voltage and current is given by Ohm's Law, which states that THE RATIO OF VOLTAGE TO CURRENT IN A METALLIC CIRCUIT IS A CONSTANT CALLED THE RESISTANCE OF THE CIRCUIT. The practical unit of resistance is called the *ohm.*

$$\frac{V}{I}=R, \qquad \text{or} \qquad \frac{\text{volts}}{\text{amperes}}=\text{ohms}$$

The voltage difference V between the ends of a resistance R in which a current I is flowing is IR.

EXAMPLE 1. Suppose our automobile battery, which gives a *PD* of 6 volts, is connected to a resistance AB such that the current flowing through the circuit is 2 amp.



a) Find the resistance of AB.*

$$\text{Resistance: } R=\frac{V}{I}=\frac{6}{2}=3 \text{ ohms}$$

b) Find the power expended by the battery:

$$\text{Power}=I\times V=2\times 6=12 \text{ watts}$$

c) Find the work done by the battery when the current flows for 10 sec.:

$$\text{Work}=QV=It\times V=2\times 10\times 6=120 \text{ joules}$$

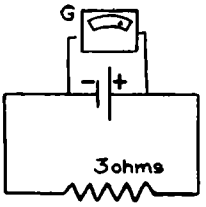
d) We can easily find how much heat is produced in the wire since we know that 4.2 joules of work done against friction produce 1 cal. of heat. Since 120 joules of work, or energy, have disappeared:

$$\text{Heat produced}=\frac{120}{4.2}=28.6 \text{ cal.}$$

Since the voltage drop V in the wire AB is equal to the resistance R of AB multiplied by the current I in AB (i.e., $V=IR$), we may write work $=QV=It\times V=It\times IR=I^2Rt$. Substitute the values of I, R, and t given in the problem in this equation and show that the work done is 120 joules.

* We shall picture resistances by a zigzag line. We have assumed here that the only resistance in the circuit is in AB. The wires connecting AB to the battery and the battery itself are assumed to have a negligible resistance.

EXAMPLE 2. *a*) Suppose that our automobile battery has an internal resistance of 1 ohm, as may happen when the battery is old. The battery still gives 6 volts when no current is flowing. If a 3-ohm resistance is connected to its terminals, the total resistance in the circuit is now 4 ohms. The battery resistance and the 3-ohm resistance are IN SERIES.

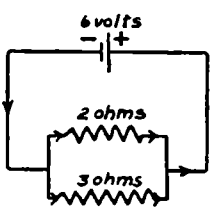


$$\text{Current flowing: } I=\frac{V}{R}=\frac{6}{4}=1.5 \text{ amp.}$$

If we placed a good voltmeter, an instrument for measuring voltage, across the terminals of the battery, the voltage recorded would be 6 volts less the voltage drop in the battery itself due to the current flowing in it. Since there is a current of 1.5 amp. flowing through the battery and the battery has a resistance of 1 ohm, the voltage drop is $IR=1.5\times1=1.5$ volts. Therefore, the *PD* across the battery,† i.e., the voltmeter reading, is $6-1.5$, i.e., 4.5 volts.

b) If the battery had an internal resistance of 2 ohms, find the current flowing through a 3-ohm resistance. (*Answer:* 1.2 amp.) Find the *PD* across the battery when this current is flowing. (*Answer:* 3.6 volts.)

c) Now suppose to a good automobile battery having no internal resistance we connect two resistances, one having a resistance of 2 ohms and the other of 3 ohms. How much current now flows through the battery and how much through each of the resistances?



There is 6 volts *PD* applied to the ends of both the 3-ohm resistance and the 2-ohm resistance. A current of V/R, i.e., $6/3=2$ amp., will flow through the 3-ohm resistance and a current of $6/2$, or 3 amp., through the 2-ohm resistance. Hence a total current of 5 amp. flows through the battery. If we had a single resistance attached to the battery such that a current of 5 amp. would flow, its resistance would be: $R=V/I=6/5=1.2$ ohms. So that the two resistances of 2 and 3 ohms placed in the manner shown are equivalent to a single resistance of 1.2 ohms.

These resistances placed in this manner are said to be IN PARALLEL. In general, if two resistances R_1 and R_2 are placed in parallel, they are equivalent to a single resistance of R ohms where

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

Let us test this formula with the data from the foregoing example:

$$R_1=3 \text{ ohms}, R_2=2 \text{ ohms}; \frac{1}{R}=\frac{1}{3}+\frac{1}{2}=\frac{2+3}{6}=\frac{5}{6}$$

$$\therefore \quad R=\frac{6}{5}=1.2 \text{ ohms}$$

Problem: Find how much current flows through an old automobile battery having an internal resistance of 1 ohm when these 2- and 3-ohm resistances are connected in parallel to it. (*Answer:* $\frac{30}{11}$ amp.) Find the *PD* across the battery when this current is flowing. (*Answer:* $3\frac{3}{11}$ volts.) Find the current in the 3-ohm coil. (*Answer:* $\frac{12}{11}$ amp.) Find the current in the 2-ohm coil. (*Answer:* $\frac{18}{11}$ amp.)

† The *PD*, or voltage which a battery gives when no current is flowing through it, is called the electromotive force (EMF) of the battery

SUMMARY

Current: $I = \frac{Q}{t}$; Amperes $= \frac{\text{coulombs}}{\text{seconds}}$

Power: $P = I \times V$; Watts = amperes $\times$ volts

Resistance: $R = \frac{V}{I}$; Ohms $= \frac{\text{volts}}{\text{amperes}}$

Work: $W = QV = It \times V$; Joules = amperes $\times$ volts $\times$ seconds.

but $V = IR$ $\therefore$ $W = It \times IR = I^2Rt$ joules

Heat produced: Heat $= \frac{W}{J}$; Calories $= \frac{\text{joules}}{4.2} = \frac{\text{amperes} \times \text{volts} \times \text{seconds}}{4.2}$

$$= \frac{It \times V}{4.2} = \frac{I^2Rt}{4.2}$$

EXAMPLE 1. Electricity comes into your home at an electrical pressure of 110 volts. The amount of current which flows in any electrical equipment depends on the resistance of the equipment. Find the current in amperes which flows in a 60-watt 110-volt lamp.

$$\text{Current: Watts} = \text{volts} \times \text{amperes; Amperes} = \frac{\text{watts}}{\text{volts}} = \frac{60}{110} \text{ amp.}$$

How much resistance does this 60-watt lamp have?

$$\text{Resistance: } R = \frac{V}{I} = \frac{110}{\frac{60}{110}} = \frac{110 \times 110}{60} = 202 \text{ ohms (approx.)}$$

Suppose a 40-watt lamp is connected to the 110-volt line. Find the current flowing in the lamp and the resistance of the lamp.

$$\text{Current: } I = \frac{\text{power}}{V} = \frac{40}{110} \text{ amp.}$$

$$\text{Resistance: } R = \frac{V}{I} = \frac{110}{\frac{40}{110}} = \frac{110 \times 110}{40} = 302.5 \text{ ohms}$$

The 40-watt lamp has more resistance than the 60-watt lamp so that the 40-watt lamp allows less current to flow through it.

EXAMPLE 2. Now if you have ever looked at your electric light bill you know that you pay by the kilowatt-hour. Find the cost of using a 60-watt lamp for 4 hr. when the rate is 10 cents per kilowatt-hour.

If the lamp burns for 1 hr., it uses up 60 watt-hours of energy, and if it burns for 4 hr., it uses up 4×60, i.e., 240 watt-hours of energy. If 1 kilowatt-hour, 1,000 watt-hours, costs 10 cents, 240 watt-hours cost $\frac{240}{1,000} \times 10$, or 2.4 cents.

EXAMPLE 3. Find the current and the cost of running an electric iron for $\frac{1}{2}$ hr., and also the total amount of heat produced when the electric iron, which has a resistance of 22 ohms, is connected to the 110-volt line. The rate is 10 cents per kilowatt-hour.

$$\text{Current flowing through the electric iron: } I = \frac{V}{R} = \frac{110}{22} = 5 \text{ amp.}$$

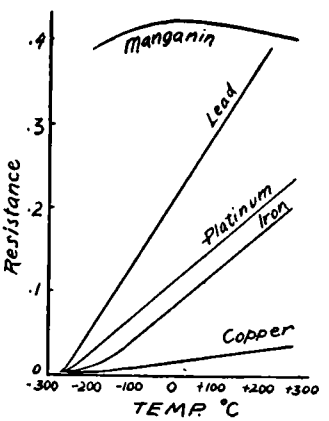
$$\text{Watts consumed: Power} = V \times I = 5 \times 110 = 550 \text{ watts}$$

$$\text{Amount of energy consumed in } \tfrac{1}{2} \text{ hr.} = 550 \times \tfrac{1}{2} = 275 \text{ watt-hours}$$

$$\text{Cost} = \frac{275}{1{,}000} \times 10 = 2.75 \text{ cents}$$

$$\text{Heat produced} = \frac{It \times V}{4.2} = \frac{5 \times 30 \times 60 \times 110}{4.2} = 236{,}000 \text{ cal.}$$

SPECIFIC RESISTANCE



THE RESISTANCE of a wire depends on its length, on its area of cross-section, and on the material of which it is made. The resistance of a wire whose length is l cm., whose area of cross-section is A sq.cm., and whose specific resistance is R' is $R = R'l/A$. THE SPECIFIC RESISTANCE OF A SUBSTANCE IS THE RESISTANCE OF A LENGTH OF 1 CM. AND AREA OF CROSS-SECTION OF 1 SQ.CM. These values are for a temperature of 20° C. The resistance of most elements increases with temperature, as shown by the graph.

TABLE OF SPECIFIC RESISTANCES IN OHMS PER CM.

	Aluminum	Brass	Copper	Silver	Gold	Carbon
R'.....	2.8×10^{-6}	8.2×10^{-6}	1.7×10^{-6}	1.5×10^{-6}	2.4×10^{-6}	$3{,}500 \times 10^{-6}$

EXAMPLE. Find the resistance at 20° C. of a piece of copper wire 100 cm. in length and 0.1 cm. in radius.

$$\text{Resistance: } R = \frac{R'l}{A}; \qquad R' = 1.7 \times 10^{-6}; \qquad l = 100 \text{ cm.}$$

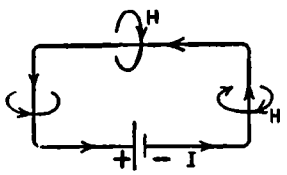
$$A = \pi r^2 = 3.14 \times 0.01 = 0.0314 \text{ sq.cm.}$$

$$\therefore \quad R = \frac{1.7 \times 10^{-6} \times 100}{0.0314} = 5.4 \times 10^{-3} \text{ ohms}$$

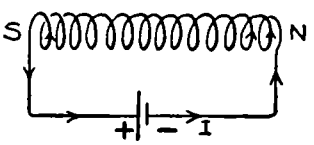
$$= 0.0054 \text{ ohms}$$

THE MAGNETIC FIELD PRODUCED BY A CURRENT

IN 1820 Oersted of Copenhagen made the important discovery that an electric current is always accompanied by a magnetic field. The direction of the magnetic field depends on the direction of the current, i.e., the flow of the electrons.* A simple way to remember this direction is to grasp the wire in the left hand, the thumb pointing in the direction of the current. The fingers will then encircle the wire conductor in the direction of the magnetic field. The strength of the magnetic field H at a point will depend on the strength of the current and the distance of the point from the wire.



If we pass a current through a helical coil of wire (shown in the diagram), the magnetic field INSIDE the coil goes from left to right. Each turn contributes to the field in this direction. This coil of wire will behave like a magnet having its N. pole on the right and its S. pole on the left.



Let I amp. of current flow through a coil which has n turns of wire per cm. We cannot offer a proof of the formula for the value of the magnetic field H, but here the formula is $H=\frac{4\pi nI}{10}$.† Suppose a piece of iron fills the inside of the coil. Then this magnetic field due to the current magnetizes the iron. We have an ELECTROMAGNET. The magnetic field is now the sum of the field H and the field due to the magnet. The total field, which we shall call B, is equal to μH: $B=\mu H$, where μ is the *permeability* of the iron. The permeability, μ, is not a constant but varies with the value of H.

EXAMPLE. A helical coil of wire with 400 turns and a length of 50 cm. carries a current of 5 amp. Find the value of the magnetic field inside the coil.

$$H=\frac{4\pi nI}{10}\ ; \qquad n=\frac{400}{50}=8 \text{ turns per cm.}; \qquad I=5 \text{ amp.}$$

$$H=\frac{4\times 3.14\times 8\times 5}{10}=50.2 \text{ gauss}$$

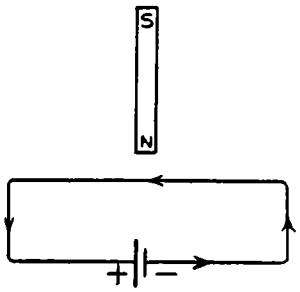
If the coil contains an iron core, the permeability μ of which for this field strength is 500, find the total magnetic field.

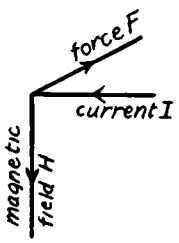
$$B=\mu H=500\times 50.2=25,100 \text{ gauss}$$

* In most texts the current in the wire is taken as flowing from the positive to the negative pole of the battery—the reverse of what we have said.

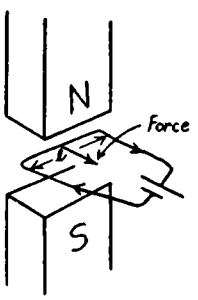
† The factor $\frac{1}{10}$ comes in due to using the practical units for current: 1 amp. is $\frac{1}{10}$ of an electromagnetic unit of current.

LET US find the force exerted by a magnetic field on a wire carrying a current. Since magnets repel or attract each other, we should expect that a wire carrying a current would also be attracted or repelled. Suppose we hold a magnet above the wire, as shown in the diagram. The current in the wire produces a magnetic field which at the N. pole of the magnet is directed perpendicularly *out* of the paper,





so that the N. pole of the magnet will be acted on by a force in this direction. By Newton's Third Law, the wire must be acted on by an equal and opposite force tending to push it *into* the paper. The magnetic field of the magnet is downward, the current is flowing across the paper, and the force on the wire is at right angles to and into the paper. These three directions are mutually at right angles, as shown in the diagram.



If a wire l cm. long carrying a current of I amp. is placed within a uniform magnetic field H and at right angles to it, the wire will be acted on by a force of $\frac{I}{10} lH$ dynes. This force will be at right angles to the field and to the length of the wire.

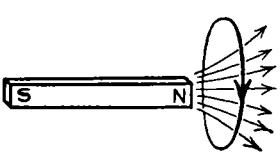
Electrical measuring instruments such as ammeters, voltmeters, etc., all work on the principle that a wire carrying a current is acted on by a force when placed in a magnetic field.

EXAMPLE. A wire 5 cm. long and carrying a current of 2 amp. is placed within and at right angles to a magnetic field whose strength is 1,000 gauss. Find the force on the wire.

$$F = \frac{I}{10} lH = \frac{2 \times 5 \times 1{,}000}{10} = 1{,}000 \text{ dynes}$$

ELECTROMAGNETIC INDUCTION

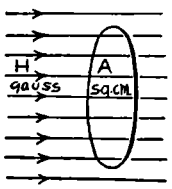
AFTER OERSTED'S discovery that an electric current produces a magnetic field there was intense activity to find if a magnetic field cannot somehow be made to generate an electric current. The "somehow" was found by



Faraday in 1831. He found that a current of electricity flows through a coil of wire whenever the magnetic field passing through the coil of wire is changed. The current lasts ONLY WHILE THE MAGNETIC FIELD IS CHANGING. If the N. pole of a magnet is quickly brought close to a coil of wire, the current flows momentarily in the direction of the arrow shown in the diagram.

The magnetic field due to the induced current *must always oppose the magnetic field* of the magnet (see problem 23).

We must now digress and follow Faraday for a short time. Faraday represented both electric and magnetic fields by *lines of force.* The direction of a line of force is the direction of the field, i.e., in the case of a magnetic field, the direction in which an isolated N. pole would move. The magnitude, or strength, of the magnetic field may be represented by the *number of lines of force cutting each sq.cm. of area at right angles to the field.* Thus in a field whose strength is 6 gauss there would be 6 lines of force cutting each sq.cm. of area at right angles to the field. If we place a coil whose area of cross-section is A sq.cm. at right angles to a magnetic field whose strength is H gauss, the total number of lines of force N passing through the coil is HA.



EXAMPLE. A coil 10 cm. in radius is placed at right angles to a field of 20 gauss. Find the total number of lines of force N passing through the coil.

$N=HA$; $A=\pi r^2=3.14\times100=314$ sq.cm.; $H=20$ gauss

$N=20\times314=6{,}280$ lines of force

WHILE WE have been talking only of the induced current, this current must be due to a *PD* induced in the circuit by a changing magnetic field. Faraday found that THE INDUCED *PD* IN A CIRCUIT IS EQUAL TO THE TIME RATE OF CHANGE OF THE TOTAL NUMBER OF LINES OF FORCE THROUGH THE CIRCUIT.

If N_0 = number of lines of force through a circuit at time $t=0$
and N_t = number of lines of force through a circuit at time t,
then N_t-N_0 = change in number of lines of force through circuit
and $\dfrac{N_t-N_0}{t}$ = time rate of change of the lines of force through the circuit.

The induced *PD* in volts* produced in the circuit is $V=\dfrac{N_t-N_0}{10^8\times t}$.

EXAMPLE 1. Find the induced *PD* in volts produced in the coil described in the previous example if it took $\frac{1}{10}$ sec. to move the coil into the field.

$$V=\frac{N_t-N_0}{10^8 t}; \qquad N_0=0; \qquad N_t=6{,}280; \qquad t=0.1 \text{ sec.}$$

$$\therefore \quad V=\frac{6{,}280-0}{10^8\times0.1}=0.000628, \text{ or } 6.28\times10^{-4} \text{ volts}$$

* The factor 10^8 comes in due to using the practical system of units. 1 volt $=10^8$ electromagnetic units of *PD*. An induced *PD* of 1 volt is produced in a single loop of wire when it cuts 10^8 lines of force per second.

EXAMPLE 2. Find the value of the current produced if the resistance of the coil described in the example above is 5 ohms.

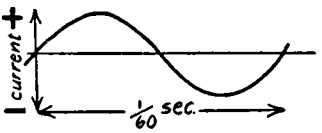
$$I=\frac{V}{R}=\frac{6.28\times10^{-4}}{5}=1.26\times10^{-4}\text{ amp.}$$

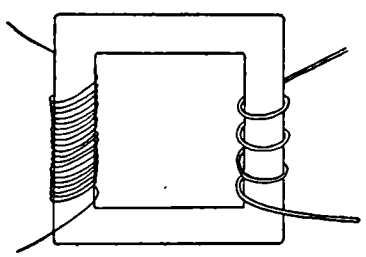
Find the total quantity of electricity which flows around the circuit.

$$\text{Quantity of electricity } Q=IT=1.26\times10^{-4}\times0.1=$$
$$=1.26\times10^{-5}\text{ coulombs}$$

ALTERNATING CURRENT AND TRANSFORMERS

THE ELECTRICITY supplied to most houses is alternating, i.e., it rushes in one direction and then in the other. It is usually 60 cycle alternating current (AC). This means that for $\frac{1}{120}$ sec. it flows in one direction and for the next $\frac{1}{120}$ sec. it flows in the other direction. Thus the magnetic field produced by the current is continually changing its direction. The great advantage of alternating current is that it can be transformed to a higher or lower voltage by a device called a *transformer*. This usually consists of two sets of coils, a primary and a secondary, wound on an iron core.





If the alternating voltage supplied to the primary coil which has n_1 turns is V_1 volts, the voltage produced at the terminals of the secondary, which has n_2 turns, is (neglecting losses) V_2, where

$$\frac{V_2}{V_1}=\frac{n_2}{n_1}$$

PROBLEMS

1. Two charges of Q_1 and Q_2 e.s.u. are placed r cm. apart in air. The force between them is 16 dynes. What would the force be between the following charges:
   a) Q_1 and $2Q_2$ placed r cm. apart in air?
   b) Q_1 and Q_2 placed $2r$ cm. apart in air?
   c) $2Q_1$ and $2Q_2$ placed $2r$ cm. apart in air?
   d) Q_1 and Q_2 placed r cm. apart in water?

2. A charge of $+5$ e.s.u. is situated at a point A.
   a) Find the strength of the electric field at a point B, 10 cm. from A.
   b) Is the field directed from A to B or from B to A?
   c) Find the magnitude and direction of the force on a charge of -20 e.s.u. placed at B.

d) Find the magnitude and direction of the force on the charge of +5 e.s.u. at A, due to the charge of −20 e.s.u. at B.

3. Can you say why a negatively charged fountain pen picks up small pieces of paper?

4. Is it possible to electrify a rod of copper by holding it in your hand and rubbing it with a piece of flannel? Give a reason for your answer.

5. On page 109 we showed how a metallic object BC supported on an insulated stand D could be negatively charged by induction from a charged rod A.
   a) What sign of charge must A have if BC is to be charged positively by induction?
   b) If the charged rod A is held near to BC while BC is charged by induction, will more, less, or the same amount of charge be induced on BC as if the charged rod A were held farther away?

   Suppose BC is electrically neutral and that a positively charged rod is drawn across BC several times:
   c) Will BC acquire a charge?
   d) If so, what sign?
   e) Does the positively charged rod lose any charge?

6. When BC is charged by induction by the charged rod A,
   a) Does A lose any charge?

   A charged object possesses energy. (Consider the energy in a lightning flash.)
   b) From where does the energy of the charge induced on BC come?

7. Is the magnetism of the N. magnetic pole that characteristic of the N. pole of a magnet or of the S. pole of a magnet?

8. The earth's magnetic field at these latitudes is about 0.4 gauss and dips at an angle of 60° to the horizontal.
   a) Find the value of the horizontal component of the earth's field.
   b) Of the vertical component.

   (The sides of a 30°, 60°, and 90° triangle are in the ratio of 1 : 1.73 : 2.)

9. A magnet has a length of 10 cm. and a pole strength of 5 units. It is placed in an E.–W. position on a frictionless horizontal table at a place where the horizontal component of the earth's magnetic field is 0.18 gauss.

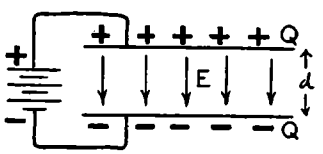
a) Will the magnet move bodily toward the N. or the S.?
b) What is the force on the N. pole of the magnet?
c) What is the resultant force on the magnet?
d) What is the moment of the force tending to cause the magnet to rotate into a N.–S. direction?

10. Would you expect it possible feebly to magnetize a bar of iron by holding it in the direction of the earth's magnetic field and then tapping it? What would be the reason for the tapping?

11. *a*) Find the magnetic field at a point 10 cm. from a N. pole whose strength is 5 units.
b) What would be the force on a unit pole placed at this point?
c) What would be the force on a pole of 20 units placed at this point?

12. The strength of each of the poles of a compass needle is 20 units and the distance between them is 10 cm. Find the magnetic field due to this magnet at a point along a line joining the poles and 25 cm. from its center.

13. A 10-volt battery is connected to two parallel metallic plates 5 cm. apart.
a) What is the value of the electric field between the plates?
b) What is the force on a charge of 1 coulomb placed in the field?



c) How many joules of work are done in moving this coulomb of charge from one plate to the other?
d) If the plates are 10 cm. apart, how much work is done in moving this coulomb of charge from one plate to the other?
e) What is the value of the electric field in this case?

14. A flashlight battery has a voltage of 3 volts and the flashlight lamp draws 0.25 amp.
a) What is the resistance of the lamp?
b) How many watts does the lamp consume?
c) If the lamp burns for 10 min., how many joules of energy have been expended?

15. A 6-volt automobile battery uses 20 amp. for 10 sec. to start the motor. How many coulombs of charge pass?

16. An electric toaster which consumes 550 watts is connected to the 110-volt line.
a) What current does the toaster take?

b) What energy is used in producing a piece of toast which takes 2 min. to brown?

c) How much does this toasting cost if the rate is 10 cents per kilowatt-hour?

d) Find how many joules there are in a kilowatt-hour.

17. An electric light and power company transmits 1,000 kw. of power to a distant town. The resistance of the line is 20 ohms. If it is transmitted at 100,000 volts,

a) What is the current carried by the transmission lines?

b) How much of this power is consumed by the resistance of the line?

If the power is transmitted at 10,000 volts,

c) What is the current carried by the transmission lines?

d) How much power is consumed by the resistance of the line?

18. A No. 201*A* radio-tube filament takes 0.25 amp. when 5 volts are applied to its ends.

a) What is the resistance of the filament?

b) How much power does the filament consume?

This 0.25-amp. current flowing through the filament causes it to emit electrons which are attracted to the plate of the tube. When the terminals of a 90-volt *B* battery are joined to the plate of the tube and the filament, a current of 2 milliamp. (0.002 amp.) flows from the filament to the plate. (Of course there is a grid in the tube, but that need not concern us.)

c) What is the filament-plate resistance of the tube?

d) How many watts are consumed in the tube?

e) How much energy does an electron have when it strikes the plate due to falling through this *PD* of 90 volts?

19. A household uses five 50-watt 110-volt lamps for 8 hr. a day.

a) At a rate of 10 cents per kilowatt-hour, how much does this cost per day?

b) When the lamps are connected to the 110-volt line, what is the current used while they are burning?

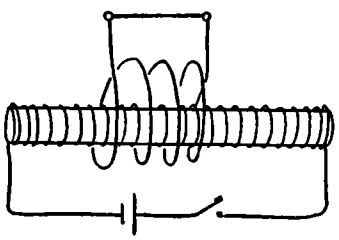
c) Are the lamps connected in series or in parallel?

d) If the lamps were connected to a 220-volt line, would they consume the same number of watts as when connected to the 110-volt line? (Remember that the resistance of the lamps is constant.)

e) What is the total amount of current used when the lamps are connected to the 220-volt line?

20. An electric heater with a coil whose resistance is 20 ohms is connected to the 110-volt line. It is immersed in a kettle of water whose mass is 1,000 gm. If the water is originally at 20° C.,
   a) How many calories of heat are necessary to bring the water to 100° C.?
   b) How many joules of energy are used?
   c) How much current does the heater take?
   d) How long is it necessary to leave the immersion heater in the water to cause the water to boil?
   e) How much does the electrical energy required to cause the water to boil cost at the rate of 10 cents per kilowatt-hour?

21. A helical coil of wire has 10 turns per cm. and carries a current of 5 amp.
   a) What is the value of the magnetic field produced?
   b) If the coil contains an iron core whose permeability is 500, find the resulting field.
   c) If the area of the core is 20 sq.cm., what is the total number of lines of force produced?



   If a coil of wire with one turn having a resistance of 2 ohms surrounds this helical coil,
   d) Find the induced *PD* produced in it when the current of 5 amp. is broken (i.e., all the lines of force removed) in 0.01 sec.
   e) Find the induced current which flows.
   f) Find the quantity of electricity which flows in the one turn.
   If the coil had one hundred turns in place of the one turn,
   g) Find the *PD* produced when the 5-amp. current is broken in 0.01 sec.

22. When a coil of wire is rotated in a room, will a *PD* be induced in it due to the earth's field?

23. In the diagram on page 121 we gave the direction of the induced current and stated that this current must produce a magnetic field which opposes the bringing closer of the magnet.
   a) Has energy had to be expended to bring the magnet toward the coil due to the repulsion of these fields?
   b) Does the energy expended account for the energy of the electric current in the wire?

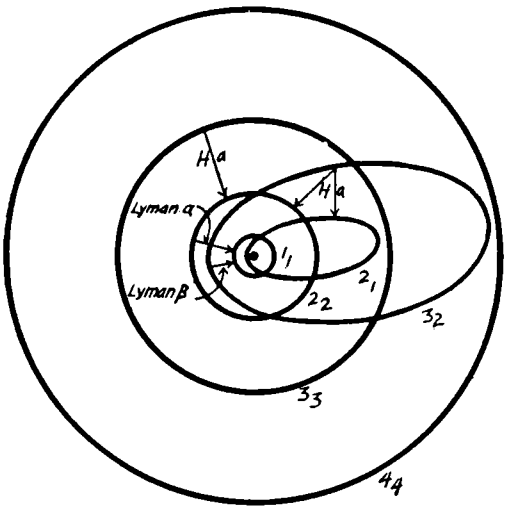
24. A wire whose length is 5 m. is stretched horizontally along the ceiling in an E.–W. direction. The wire is carrying a current of 1 amp. Find the total force on the wire due to the earth's field, which is 0.4 gauss.

25. A wire carrying a current is held so that it is in line with the earth's magnetic field. Will the wire be acted on by a force due to the magnetic field of the earth?

26. Christmas tree lamps are rated to take a *PD* of about 8 volts.
    a) When these lamps are connected to the 110-volt line, are they connected in series or in parallel?
    b) How many of these lamps would there be in one 110-volt plug connection?

    If each lamp takes 0.25 amp.,
    c) How many watts does each consume?
    d) What is the resistance of one lamp?
    e) If one of these lamps were connected directly to the 110-volt line, what would happen to it?

CHAPTER 11



CONDUCTION OF ELECTRICITY IN LIQUIDS AND GASES

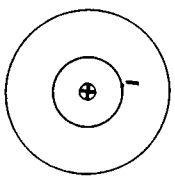
IN THE discussion of the kinetic theory of gases we thought of molecules as hard elastic spheres moving around with a high velocity. This picture of the atom worked very well for interpreting the pressure, temperature, etc., of a gas. Toward the beginning of the present century, experiments on the discharge of electricity through gases showed that atoms and molecules could be broken down still further, into positive and negative parts. The negative parts, no matter from what atom they are derived, are always the same and are called ELECTRONS. These electrons have a very small mass, about $\frac{1}{1,846}$ of the mass of the lightest atom known—the hydrogen atom. Lord Rutherford showed that the mass of the atom is associated with the positively charged part of the atom called the NUCLEUS. Experiments further disclosed that the size of the electrons and the nucleus were very small, indeed, compared to the whole atom. From these experiments a pictorial representation of an atom was offered by Rutherford and Bohr.

RUTHERFORD-BOHR ATOM

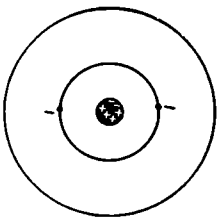
THE NUCLEUS of the atom is surrounded by electrons which are arranged in *shells*. The atom has been compared to a small solar system, the nucleus as the sun at the center and the electrons as the planets.* Let us look at a few atoms.

* The electrons are supposed to be rotating about the nucleus at high speeds.

Hydrogen has an atomic weight of 1 and a positive charge of 1.† Since an atom must be electrically neutral, the hydrogen atom must have one electron outside the nucleus. This electron is called a *planetary electron.* The nucleus of the hydrogen atom has a mass 1 and a charge +1 and is called a PROTON. Another entity in atoms is the neutron. The neutron was discovered by Chadwick in 1932 and, as its name implies, is electrically neutral. The mass of the neutron is slightly larger than that of a proton, though for approximate purposes we may say that both have a mass of unity, i.e., 1 atomic unit. The nuclei of atoms are considered to be built up of protons and neutrons. Thus the building blocks of atoms are protons, neutrons, and planetary electrons.



Hydrogen atom



Helium atom

The next lightest atom after hydrogen is helium. This has an atomic weight of 4 and a positive charge of 2 on its nucleus. The nucleus of the helium atom consists, then, of 2 protons and 2 neutrons and is called an ALPHA PARTICLE, α. Thus the helium atom is built up of a nucleus (mass 4, charge +2) of exceedingly small dimensions surrounded by 2 planetary electrons at a relatively great distance away. The resultant positive charge on the nucleus, numerically equal to the number of planetary electrons, is called the ATOMIC NUMBER of the atom.

Let us consider the heaviest atom occurring in nature, uranium, of atomic weight 238 and atomic number 92. The nucleus may be considered to be built up of 92 protons and 146 neutrons, giving an atomic number of 92 and an atomic weight of 238. There are, then, 92 planetary electrons in the uranium atom.

Now these electrons are not arranged haphazardly, but in shells, the number in each shell being quite definite. The one planetary electron of hydrogen goes into the shell nearest the nucleus. Proceeding to helium, the evidence shows that both its planetary electrons go into the first shell. This first shell is then complete and can hold no more. In the case of lithium, with 3 planetary electrons, 2 go into the first shell and the third into the second shell. This second shell is complete when it has 8 electrons. In the accompanying table the electronic structure of the lighter elements is given.‡

† The unit charge we are using here is the charge on the electron, 1.60×10^{-19} coulombs. An excellent account of the method used by Professor Millikan for determining the charge on the electron is given in his book, *The Electron.*

‡ This is a modification of the Periodic Table originally developed by the Russian chemist, Mendeléeff. For the complete table see Lemon, *From Galileo to the Nuclear Age.*

Group	0	I	II	III	IV*a*	V*a*	VI*a*	VII*a*	VIII*a*			Ia	IIa	IIIa	IV	V	VI	VII	0 Repeated
First series	4.0 2 He 2,0,0,0	6.9 3 Li 2,1,0,0	9.0 4 Be 2,2,0,0	10.8 5 B 2,3,0,0											12.0 6 C 2,4,0,0	14.0 7 N 2,5,0,0	16.0 8 O 2,6,0,0	19.0 9 F 2,7,0,0	20.2 10 Ne 2,8,0,0
Second series	20.2 10 Ne 2,8,0,0	23.0 11 Na 2,8,1,0	24.3 12 Mg 2,8,2,0	27.0 13 Al 2,8,3,0											28.1 14 Si 2,8,4,0	31.0 15 P 2,8,5,0	32.1 16 S 2,8,6,0	35.5 17 Cl 2,8,7,0	39.9 18 A 2,8,8,0
Third series	39.9 18 A 2,8,8,0	39.1 19 K 2,8,8,1	40.1 20 Ca 2,8,8,2	45.1 21 Sc 2,8,8,3	48.1 22 Ti 2,8,8,4 or 2,8,9,3	51.0 23 V 2,8,8,5 or 2,8,10,3	52.0 24 Cr 2,8,8,6 or 2,8,11,3	54.9 25 Mn 2,8,8,7 or 2,8,13,2	55.8 26 Fe 2,8,13,3 or 2,8,14,2	58.9 27 Co 2,8,14,3 or 2,8,15,2	58.7 28 Ni 2,8,15,3 or 2,8,16,2	63.6 29 Cu 2,8,18,1 or 2,8,17,2	65.4 30 Zn 2,8,18,2	69.7 31 Ga 2,8,18,3	72.6 32 Ge 2,8,18,4	75.0 33 As 2,8,18,5	79.2 34 Se 2,8,18,6	79.9 35 Br 2,8,18,7	82.9 36 Kr 2,8,18,8

Atomic Weight — Atomic Number

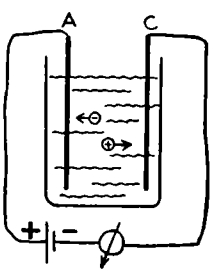
ELEMENT

Electron Arrangement

If you know any chemistry, you realize that the elements in a vertical column constitute a group of elements which have very similar chemical properties. The zero (0) group contains all the inert gases. These have no chemical activity: they do not combine with other elements to form compounds. These elements all have *completed* shells. The elements in Group I all have 1 electron outside of the completed shells, while elements in Group II have 2 electrons outside the completed shells, etc.

From chemical considerations we may assume that chemical activity is due to the tendency of atoms to achieve an arrangement of their planetary electrons similar to that of the inert gas nearest in atomic number. Sodium, atomic number 11 (2, 8, 1), has a tendency to lose 1 electron, leaving a residue with a resultant positive charge of 1. This residue is called the *sodium ion*, Na^+. Chlorine, atomic number 17 (2, 8, 7), tends to assume an argon-like structure by taking on 1 electron, thus producing the negative chloride ion, Cl^-. Elements in Group II have a tendency to give off 2 electrons, forming doubly charged positive ions (as Mg^{++}), while elements of Group VI have a tendency to take on 2 electrons, producing doubly charged negative ions.

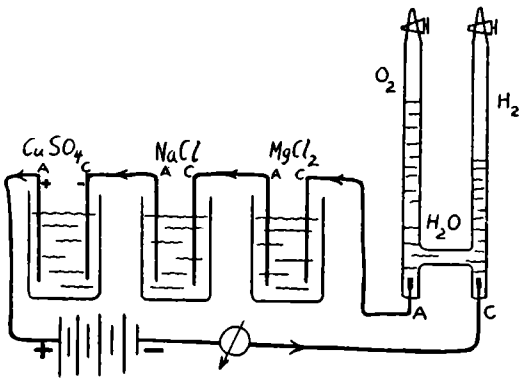
Let us return to the elements sodium and chlorine. The salt, sodium chloride, is formed from Na^+ and Cl^- ions held together by electrostatic attraction. When wires connected to a battery are placed in solid sodium chloride (NaCl), no current flows. The ions are not free to move. If the sodium chloride is heated so that it becomes molten, or if it is dissolved in water, the ions are free to move and can conduct a current.



The diagram represents two wires, *A* and *C*, placed in molten NaCl. (The heating element is not shown.) The two wires which dip into the molten NaCl are called *electrodes*. *A*, connected to the positive pole of the battery, is called the *anode*, while *C*, connected to the negative pole, is called the *cathode*. The Na^+ ions travel toward the cathode while the Cl^- ions travel toward the anode. If the electrodes are made of suitable material—platinum, for instance—the ions will be neutralized at the respective electrodes and give rise to sodium and chlorine atoms. The passage of an electric current through a conducting solution is rendered possible by the presence of ions and the process is called ELECTROLYSIS.*

* While this material is somewhat out of our field, we are introducing it here to interpret Faraday's Laws of Electrolysis. These results would hardly be intelligible without a fair knowledge of chemistry unless we adopt some such scheme as this.—*Author's apology.*

LET US now pass the same current through each of a number of cells. We shall use copper sulphate $CuSO_4$, magnesium chloride $MgCl_2$, both of these dissolved in water, molten sodium chloride, and slightly acidified water, H_2O.† After a current has flowed for a suitable time, we shall determine the amount of each substance deposited or liberated. We shall find that for every 96,500 coulombs of electricity passed through the solution, the following number of grams of the different substances involved have been deposited or liberated:



Copper (Cu)	Sodium (Na)	Chlorine (Cl_2)	Oxygen (O_2)	Hydrogen (H_2)
$\frac{63.57}{2}$	$\frac{23.00}{1}$	$\frac{35.46}{1}$	$\frac{16.00}{2}$	$\frac{1.008}{1}$

The atomic weight of copper is 63.57 and that of sodium is 23. Now 1 gm. atomic wt. (the weight in grams equal to the atomic weight) of all elements contains the same number of atoms. So that 63.57 gm. of copper contains the same number of atoms as 23 gm. of sodium or 16 gm. of oxygen, namely, 6.02×10^{23}. Thus the 96,500 coulombs of electricity have been carried by 3.01×10^{23} ions of copper, 6.02×10^{23} ions of sodium, or 3.01×10^{23} ions of oxygen. If e denotes the magnitude of the charge on the sodium ion, 6.06×10^{23} ions have a total charge of 96,500 coulombs: $6.02\times10^{23}\,e = 96{,}500$ coulombs. Each copper ion must have a charge of $2e$. Let us see what the magnitude of this charge e is:

$$e=\frac{96{,}500}{6.02\times10^{23}}=1.60\times10^{-19}\text{ coulombs}$$

This is the charge on an electron and indicates that the atoms have become ions by the gain or loss of one or more electrons.

All this is exactly what we should expect from our atomic picture. The copper atom (2, 8, 17, 2) in becoming a copper ion gives off its two outer electrons to produce an ion Cu^{++}. Oxygen (2, 6) takes on two electrons, producing a doubly charged negative ion.

† We shall, of course, overlook any practical difficulties which might arise in such a set-up.

It is customary to denote the number of electrons which an atom takes on or gives off when it becomes an ion by the term *valence*. Thus the valence of sodium or chlorine atoms is 1. Atoms in Group II or Group VI are divalent. We may state Faraday's Law of Electrolysis as: **96,500 coulombs of electricity deposits or liberates in electrolysis a weight in grams equal to the atomic weight of any element divided by its valence.**

EXAMPLE. A steady current of 4 amp. flows for 10 min. through a solution of silver nitrate. Find how much silver is deposited on the cathode.

Silver (Ag) has an atomic weight of 108 and is univalent, so that silver ions have a single positive charge (Ag^+).

Number of coulombs of electricity which pass through the solution

$$= It = 4 \times 10 \times 60 = 2{,}400 \text{ coulombs}$$

The number of grams of any element liberated by 96,500 coulombs is equal to its atomic weight divided by its valence, so that

96,500 coulombs liberate 108 gm. of silver

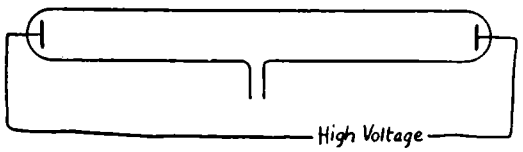
$$2{,}400 \text{ coulombs liberate } 108 \times \frac{2{,}400}{96{,}500} = 2.69 \text{ gm. of silver}$$

CONDUCTIVITY OF GASES

GASES AT ordinary temperatures and pressures are very good insulators. Otherwise, how would it be possible for electric-light companies to transmit electricity at voltages as high as 200,000 volts? Yet, as you know, air does at times conduct a current. This happens every time a lightning flash occurs. We can imitate a lightning flash on a small scale by connecting two pointed electrodes placed fairly close together to a potential difference as low as 20,000 volts. It is not so much the voltage as the electric field produced by the voltage which causes a current to flow in the air. Air always contains a few ions. These are produced by cosmic rays or radioactive matter which remove electrons from the oxygen and nitrogen molecules in the air. These electrons immediately attach themselves to a neutral molecule, producing a negative ion. Of course the positive and negative ions are continually recombining, but then the cosmic rays are always producing new ions.

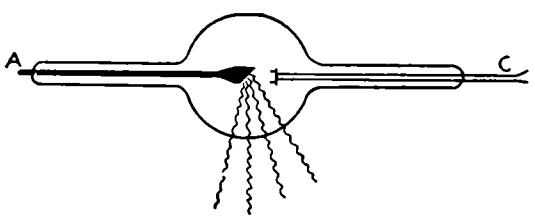
These positive and negative ions are accelerated by the electric field and thus acquire additional energy. However, before an ion travels very far it will bump into a neutral molecule. If it has acquired enough energy, it may be able to remove an electron from the neutral molecule. Thus a new ion is formed which in its turn acquires energy due to the intense electric field and is able to produce further ionization. In this way the electric field produces *multiple ionization*. The motion of the ions formed constitutes the current through the air.

Suppose we now seal our electrodes into a glass tube and pump some of the air out. As the pressure is reduced, the number of molecules per cubic centimeter is reduced, and hence the farther an ion has to travel before it hits a neutral molecule. Thus at low pressures the ion acquires more energy and multiple ionization can more readily take place than at high pressures. If we reduce the pressure in the tube to about one-ten-thousandth of an atmosphere, any electrons removed from the molecules by an impact will have a good chance of traveling down the tube without any collisions. These electrons are called *cathode rays* because they start from the vicinity of the cathode. If these cathode rays strike the glass walls, they produce a greenish color in the glass called *fluorescence.*

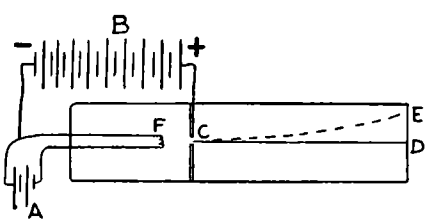


Roentgen showed that when cathode rays are stopped suddenly, they produce X-rays. By using a good high vacuum pump it is possible to reduce the pressure of the air in the tube so much that no current passes even with a very high voltage. There are still molecules present, but they are so few that ionization does not take place.

Electrons can be produced in other ways than by the ionization of a gas. Edison discovered that a wire heated red hot gives off electrons. Coolidge used this principle in the X-ray tube which bears his name. The Coolidge tube is evacuated to such a high degree that no cathode rays are produced. Electrons produced by heating a filament are accelerated by a high voltage toward a target. The electrons, being stopped suddenly, generate X-rays in the target. Still another way to produce electrons is to allow ultra-violet light to fall on a clean zinc plate. The atoms of zinc are ionized by the action of the ultra-violet light. Electrons, called in this case *photoelectrons*, are given off. X-rays are even more effective than ultra-violet light in ejecting photo-electrons from metals.



EXAMPLE. Determination of the mass of an electron: The apparatus shown alongside is to be our balance for weighing an electron. The electron is so light that the most delicate chemical balance would be utterly useless. An understanding of this peculiar balance will require all our knowledge of mechanics and electricity. Let us first describe our apparatus:



A highly evacuated tube has a filament F and a small hole in a metal plate at C. The battery A heats the filament and causes it to give off electrons which are accelerated toward C

by the battery B. Electrons will shoot through the hole C into the space beyond, where they enter a magnetic field which has its lines of force at right angles to the plane of the paper.* (The magnetic field is not shown in the diagram.) Are its lines of force directed into or out of the paper to cause the electrons to be deflected upward?§

Here are the experimental data from which we are to obtain the mass of the electron:

Voltage of battery $B = 500$ volts; strength of magnetic field $= 3$ gauss

The electrons are bent into an arc of a circle 25 cm. in radius. (This latter quantity has been obtained from the distance CD and the deflection DE.)

Charge on an electron $= 1.60 \times 10^{-19}$ coulombs; its mass we shall call m

First let us find the energy which the electrons have when they pass through the hole C: A charge Q coulombs falling through a PD of V volts acquires an energy of VQ joules.

$$VQ = \tfrac{1}{2}mv^2$$

Since $\frac{1}{2}mv^2$ is in ergs (m in gm., v in cm. per sec.), we must change VQ from joules to ergs.

$$VQ = 500 \times 1.60 \times 10^{-19} = 8.00 \times 10^{-17} \text{ joules} = 8.00 \times 10^{-10} \text{ ergs}$$

$$\therefore \tfrac{1}{2}mv^2 = 8.00 \times 10^{-10} \text{ ergs}$$

If there is no magnetic field present, the electrons travel straight on to D, but when the field is present, they are bent into an arc of a circle. Do you recall from the previous chapter that the force on a wire l cm. long carrying a current of I amp. placed at right angles to a field of H gauss is $HIl/10$ dynes? These electrons traveling toward D constitute a current of electricity ev, where e is the charge on the electron. The force exerted by the magnetic field on the electrons is $Hev/10$ dynes. It always acts at right angles to the direction in which the electrons are moving. (Now in mechanics we had a problem similar to this of a boy swinging a stone around in a circle. The tension in the string, the centripetal force, caused the stone to move in a circle.) The force of $Hev/10$ dynes causes the electrons to move in a circle.

$$\therefore \frac{Hev}{10} = \frac{mv^2}{r}$$

Canceling v on each side of the equation, we get

$$\frac{He}{10} = \frac{mv}{r}$$

$$mv = \frac{Her}{10} = \frac{3 \times 1.00 \times 10^{-19} \times 25}{10} = 75 \times 1.60 \times 10^{-20} = 120 \times 10^{-20} \text{ gm. } \times \text{cm. per sec.}$$

We have already found that $\frac{1}{2}mv^2 = 8.00 \times 10^{-10}$ ergs. From this and the last equation we can find m, the mass of the electron. This requires a little juggling, but it is not very difficult.

$$mv = 120 \times 10^{-20}.$$

Squaring both sides of the equation:

$$m^2v^2 = 120 \times 120 \times 10^{-40} = 14.4 \times 10^{-37}$$

*Does the energy and velocity of the electrons at C depend on the distance of the filament F from C? When there is no magnetic field, is the velocity of the electrons constant as they go from C to D? (*Answer:* No; yes.)

For m^2v^2 we may write $\frac{1}{2}mv^2 \times 2\ m$.

$$\therefore \quad \tfrac{1}{2}mv^2 \times 2m = 14.22 \times 10^{-37}$$

$$\text{or} \quad 7.95 \times 10^{-10} \times 2m = 14.22 \times 10^{-37}$$

$$\therefore \quad m = \frac{14.22 \times 10^{-37}}{2 \times 7.95 \times 10^{-10}} = \frac{14.22 \times 10^{-27}}{15.90}$$

$$= 8.9 \times 10^{-28} \text{ gm.}$$

The accepted value for the mass of an electron is 9.01×10^{-28} gm., so our apparatus has given us quite a good value for its mass.

POSITIVE IONS: The experiment we have just described could have been performed with cathode rays. When cathode rays are produced in a gaseous discharge tube positive ions are also present. These positive ions are atoms or molecules of the gas minus one or more electrons. The ions travel toward the cathode. A very fine hole in the cathode allows the positive ions to pass through it where they enter a highly evacuated chamber. By analysis with electric and magnetic fields their masses may be obtained. Naturally you would expect that we should get nothing new, but just verify the atomic-weight table. Actually, when these experiments were performed by Sir J. J. Thomson, some very surprising results were obtained. We shall give as an example the results on chlorine gas, atomic weight 35.46. It was found that chlorine gas consists of a mixture of atoms which have masses of 35 and 37. Why these two kinds of atoms should always be mixed in the same proportion, so as to give an average of 35.46, is as yet unanswered. These atoms both have an atomic number of 17. Since the chemical properties of an atom are determined solely by the number of its planetary electrons (17 in this case), it is obvious that these two atoms can never be separated by chemical methods. Chlorine is by no means an exception among the elements. In fact, about nine-tenths of the elements have been found to be a mixture of several kinds of atoms. For any individual element these different atoms have the same atomic number but a different mass or atomic weight. The name *isotope* is given to such atoms. (How many protons and neutrons does the nucleus of the chlorine isotope mass 35 contain? How many for the chlorine isotope mass 37?)§

RADIOACTIVITY: The nuclei of most atoms are extremely stable, but there are some of the heavier elements whose nuclei break up spontaneously. These are called *radioactive* atoms. One type of radioactive disintegration consists in the ejection of an *alpha* (α) *particle* from the nucleus with a veloc-

§ Answer to question on the previous page: Out of the paper.

ity which may amount to one-tenth of the velocity of light. You will recall that an α-particle is the nucleus of a helium atom. Another type of disintegration is that in which an *electron* is ejected from the nucleus with a velocity which, in this case, may be within a small fraction of the velocity of light. Electrons ejected from the nucleus in this manner are called *beta* (β) *particles.* The beta particles are considered to come from the nucleus by a neutron changing into a proton in the nucleus. The emission of these two kinds of particles is often accompanied by a radiation resembling light but of very much shorter wave-length. This radiation which travels with the same velocity as light, 186,000 mi. per sec., or 3×10^{10} cm. per sec., is known as *gamma* (γ) *radiation.*

The effect of the ejection of an α-particle (mass 4, charge +2) from a nucleus will be the reduction of the nuclear charge of the nucleus by 2 and of the mass by 4. There will be produced a new atom having an atomic number 2 units lower and an atomic weight 4 units lower than the parent-atom. Two planetary electrons will be lost from the atom because the positive charge on the nucleus is no longer able to hold them. The ejection of a β-particle (mass negligible, charge −1) will increase the nuclear charge by 1, but the atomic weight of the new atom will be the same as that of the parent-atom. We can summarize these results:

	Mass	Charge
α-particle	4	+2
β-particle	negligible	−1
γ-ray	negligible	0

Let us follow the uranium atom, U I, as it disintegrates into radium, Ra. U I first gives off an α-particle producing UX_1, etc. We see that U I and

	U I	$\xrightarrow{\alpha}$	UX_1	$\xrightarrow{\beta}$	UX_2	$\xrightarrow{\beta}$	U II	$\xrightarrow{\alpha}$	Ionium	$\xrightarrow{\alpha}$	Ra
Atomic number	92		90		91		92		90		88
Atomic weight	238		234		234		234		230		226

U II are isotopes and so are UX_1 and ionium. The rate at which these disintegrations take place is very different for the different atoms. Thus, if we have a million atoms of U I, half of these will be changed to UX_1 in about a billion years, while in the case of a million atoms of UX_2, half of them will be changed to U II in about a minute.

When α, β, and γ rays are shot into a gas, they ionize it, i.e., they eject electrons from the atoms or molecules of the gas. The α- and β-particles ionize all along their path. Hence their energy is soon spent, so that they do

§ Answer to questions on the previous page: 17 protons, 18 neutrons; 17 protons, 20 neutrons.

not travel more than a few centimeters in a gas at atmospheric pressure. On the other hand, γ-rays travel many meters in air.

Alpha-particles with their relatively large mass and high velocity possess enormous energy and are capable of tearing right through the planetary electrons of atoms. If they happen to be headed straight for the nucleus of an atom, they may be repelled by the positive charge of the nucleus, but when this is not too great they may penetrate the nucleus. The effect of a direct hit of an α-particle with the nucleus of most light atoms is to cause the nucleus to eject a proton. Whether a proton is ejected or not depends not only on the energy of the α-particle but also on the stability of the nucleus with which it collides. Protons are ejected more easily from nuclei of atoms whose atomic number, i.e. nuclear charge, is odd than from those of even atomic number. In this connection it is interesting to note that the surface rocks of the earth are mostly made up of atoms having even nuclear charges. Such atoms are apparently stable.

Let us consider an experiment performed by Lord Rutherford in which α-particles were shot into nitrogen gas. We must remember that only one α-particle in several hundred thousand penetrates into the nucleus. Nitrogen has an atomic weight of 14 and atomic number 7. This we shall represent by N_7^{14}. The α-particle making a direct hit actually joins with the nucleus with the subsequent ejection of a proton. An α-particle will be represented by a_2^4, and a proton by p_1^1. A new atom is produced, of atomic weight 17 and atomic number 8, i.e., O_8^{17}. This is an isotope of oxygen. The equation representing this transformation may be written:

$$N_7^{14} + a_2^4 \rightarrow O_8^{17} + p_1^1$$

Notice that the total atomic weight, and the total nuclear charge, must balance on both sides of the equation. This is an example of artificial disintegration. The number of O_8^{17} atoms is, of course, exceedingly small.

Another interesting example of artificial disintegration is that of the collision of an α-particle with a beryllium nucleus. The new particle produced is is known as a NEUTRON. It has the same mass as a proton but no charge. This is possibly a proton combined with an electron. We shall represent the neutron by n_0^1. Beryllium of atomic weight 9 and atomic number 4 will be Be_4^9. The equation representing this change is

$$Be_4^9 + a_2^4 \rightarrow C_6^{12} + n_0^1$$

Atomic nuclei besides being broken up by α-particles can also be broken up by protons, which have a very high velocity due to a powerful electrical field.

An interesting example of this is the collision of a proton with one of the isotopes of lithium of atomic weight 7 and atomic number 3. The result of this collision is the production of two α-particles:

$$Li_3^7 + p_1^1 \rightarrow \alpha_2^4 + \alpha_2^4$$

There is still another particle called the POSITRON which was discovered by Anderson in California. This is a positive electron, i.e., it has the same mass as an electron but a positive instead of a negative charge.

PROBLEMS

1. How many grams of hydrogen and of oxygen will 96,500 coulombs of electricity liberate from slightly acidified water by electrolysis?

2. A steady current of 0.5 amp. is passed through slightly acidified water for 32 min. and 10 sec. The resulting hydrogen and oxygen are collected.
   a) How many coulombs of electricity have flowed?
   b) Find the mass of hydrogen and of oxygen liberated.
   c) Find the number of molecules of hydrogen and oxygen liberated.
   d) What is the charge carried by a hydrogen ion? By an oxygen ion?

3. In the electrolysis of molten sodium chloride (NaCl) 11.5 gm. of sodium are deposited.
   a) How many coulombs of electricity were used?
   b) How many grams of chlorine gas are liberated?
   c) How many cubic centimeters of chlorine gas at STP are liberated?
   d) How many sodium ions were discharged?
   e) How many chlorine ions were discharged?
   f) Is the sodium deposited at the anode or the cathode?

4. From the table of the atomic structures of the lighter elements (p. 131) find
   a) The charge on the magnesium (Mg) ion in solution.
   b) The charge on the bromine (Br) ion in solution.
   c) The charge on the aluminum (Al) ion in solution.

5. How many grams of the following will be deposited or liberated in electrolysis by 96,500 coulombs:
   a) Magnesium? *b*) Bromine? *c*) Aluminum?

6. How many atoms have been deposited or liberated
   a) Of Magnesium? *b*) Of bromine? *c*) Of aluminum?

7. A spoon is silver-plated by electrolytic methods.
   a) Will the spoon be the anode or the cathode?

Silver nitrate is a solution used for the electroplating and a bar of silver is the other electrode. The spoon has a surface area of 20 sq.cm. A coating of silver 0.001 cm. is plated on. The density of silver is 10.5 gm. per cc.

b) How many grams of silver are deposited?

c) How many coulombs of electricity pass through the solution?

d) If a current of 0.1 amp. is used, how long must it flow?

8. A beam of cathode rays is produced in a tube containing air at a low pressure. A voltage of 10,000 volts is used.

   a) Find the energy of a cathode ray, assuming it acquires the full energy due to falling through this potential difference.

   b) If hydrogen were used in the tube in place of air, would the cathode rays acquire the same energy due to this potential difference?

9. If positive rays are produced in hydrogen using a voltage of 10,000 volts, find the energy which the hydrogen ions would acquire.

10. Find the mass of the hydrogen atom. (Recall that the mass of the hydrogen atom is 1,850 times that of the electron.)

11. Find the approximate velocity of the hydrogen ions described in problem 9.

12. Neon, atomic weight 20.2, is found to be made up of isotopes having masses of 20 and of 22. If positive rays of neon are formed, using 10,000 volts,

   a) What is the energy which these ions acquire?

   b) What velocity does the isotope whose mass is 22 acquire?

   (Do not go through a lot of arithmetic, but see if you can work the problem by inspection. $\sqrt{22} = 4.7$.)

13. Find the force due to the magnetic field of 10 gauss acting on the neon isotope whose mass is 22 when the voltage applied to the tube is 10,000 volts. Use the value for the velocity of the ion found in problem 12*b*.

14. This is a problem in simple proportion: Find how many atoms of mass 20 and how many of mass 22 there are in a vessel which contains 100 atoms of neon of atomic weight 20.2.

15. When fluorine is bombarded with neutrons, an isotope of nitrogen is produced and a particle is shot out. Can you say from the following equation what the particle will be?

$$F_9^{19} + n_0^1 \rightarrow N_7^{16} + ?$$

16. If you enjoy filling in these equations, here is another:

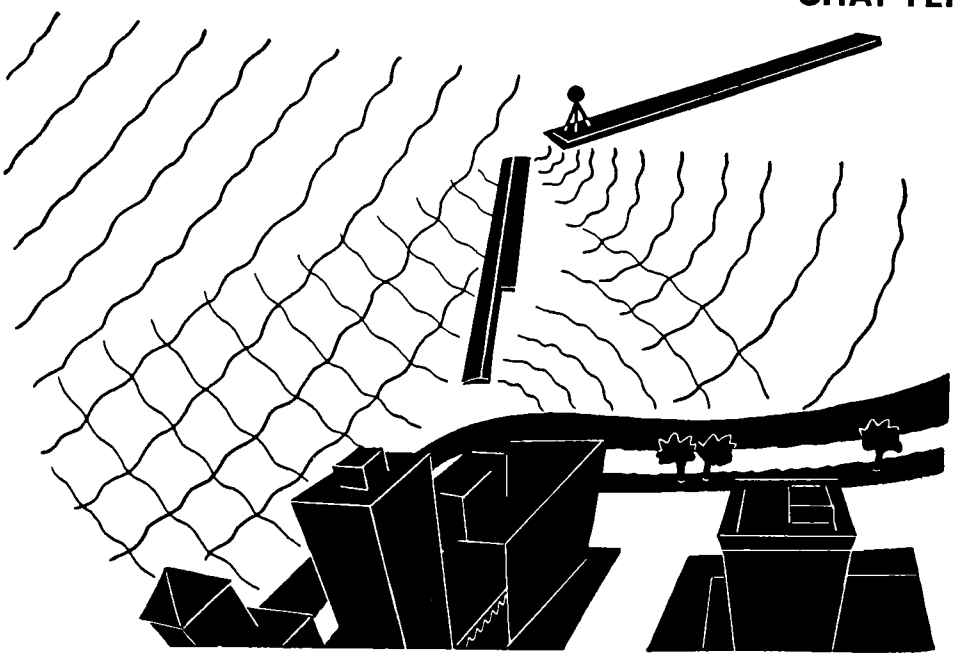
$$Be_4^9 + p_1^1 \rightarrow Li_3^6 + ?$$

17. Thorium goes through a long series of disintegrations, finally producing ThD, i.e., lead. Trace this disintegration, giving the atomic number and atomic weight of the various atoms produced. Thorium (Th) has an atomic number of 90 and an atomic weight of 232.

$$Th \xrightarrow{\alpha} MsTh1 \xrightarrow{\beta} MsTh2 \xrightarrow{\beta} RaTh \xrightarrow{\alpha} ThX \xrightarrow{\alpha} Th \xrightarrow{\alpha} ThA \xrightarrow{\alpha} ThB \xrightarrow{\beta} ThC \xrightarrow{\beta} ThC' \xrightarrow{\alpha} ThD$$

18. *a*) What atom is an isotope of Th?
    b) What atom is an isotope of ThD?

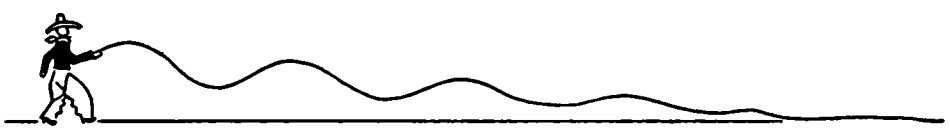
19. *a*) Are α-particles deflected by a magnetic field?
    b) Are β-particles deflected by a magnetic field?
    c) Are γ-rays deflected by a magnetic field?



A bit of Chicago's lake front from the Palmolive Building

WAVE MOTION

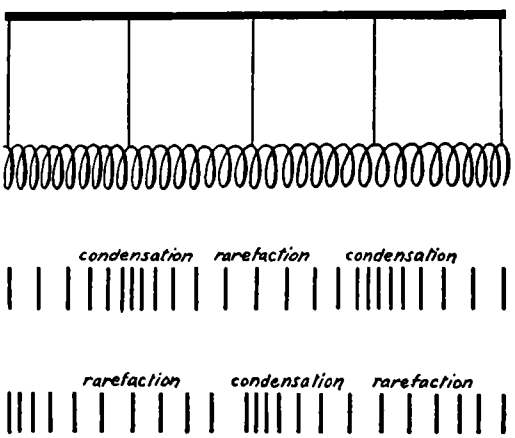
WE MAY all be able to recognize a wave motion when we see it, yet it is probable that few of us could give a physical description of what is happening. Waves are elusive creatures; they never stand still to give us a sporting chance at analyzing them. What is it about a wave that moves forward?



Let us look at the person shaking the rope in a regular manner and sending a series of waves down it. Certainly the rope itself does not move forward. It is the *wave form* that moves forward. *Crests and troughs* started by his hand move along the rope. We may say that WAVE MOTION IS THE TRANSMISSION OF A STATE OF VIBRATION THROUGH SOME MEDIUM. This transmission of vibrations means that energy is being transmitted. The far end of the rope lying on the ground eventually vibrates and thus acquires energy when the waves reach it.

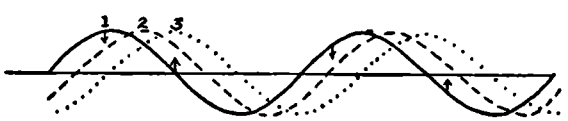
What is the nature of the vibrations in this case? Let us fix our attention on any small portion of the rope, or, better still, on a particle in the rope. We

see that the particle vibrates regularly up and down, i.e., in a direction *perpendicular* to the direction in which the wave is traveling. Such a wave motion is called TRANSVERSE. Now all that is necessary for a wave is that the particles of which the medium is composed transmit vibrations in a regular manner. Exactly how they vibrate with reference to the direction in which the wave is traveling gives us the various types of waves. One type which might immediately suggest itself is that in which the particles vibrate to and fro *in the same direction* as the wave is traveling. We have then a LONGITUDINAL wave. This we can illustrate by giving the end of a coiled spring suspended in a horizontal line a series of regular vibrations in the direction of its length. Any particle in the spring will now vibrate horizontally to and fro, producing *compressions* and *rarefactions* which travel down the spring just as the crests and troughs moved down the rope. A third type of waves is shown by water waves produced on the surface of smooth, uniformly deep water. In this case the water particles move round and round in a vertical circle. This motion gives crests and troughs, but they are not of quite the same shape as those of the transverse waves in the rope.

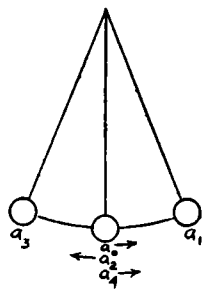


All the wave types mentioned have common characteristics. The wave form travels forward in the medium by virtue of some connection between the particles of the medium. Also all the particles of the medium vibrate to and fro about some mean position with the same frequency and do *not* travel with the wave.

LET US now analyze in some detail a wave motion; for example, a transverse wave in a rope. The diagram shows three pictures of the rope taken at intervals as the wave travels to the right. Picture 2 was taken a short time after Picture 1, and during this time the crest has moved to the right. The particles of which the rope is composed have moved up or down according to their position on the rope. Compare the motion of any of these particles with the vibration of a pendulum. The particles vibrate to and fro about their mean positions (the horizontal central line of the



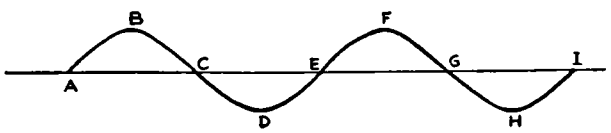
wave) just as the pendulum vibrates about its mean position a_0. The pendulum bob starts out from a_0, comes to rest at a_1, then back to a_2, is again at rest at a_3, and then returns to a_4, repeating this complete vibration indefinitely. THE TIME TAKEN FOR ONE COMPLETE VIBRATION IS CALLED THE PERIOD τ. It is also the time taken by the bob to go from a_1 to a_3 and back to a_1. What fraction of the period τ does it take for the bob to go from a_0 to a_1?§ THE FREQUENCY OF A VIBRATION, n, IS THE NUMBER OF COMPLETE VIBRATIONS PER SECOND. We can find the relation between τ and n quite easily. Suppose the pendulum bob, or the particles of the rope, execute one complete vibration in $\frac{1}{4}$ sec. They will then make 4 complete vibrations per sec. Thus



$$\text{Frequency} = \frac{1}{\text{period}}; \qquad n = \frac{1}{\tau} \text{ and } \tau = \frac{1}{n}$$

THE MAXIMUM DISTANCE WHICH A PARTICLE VIBRATES FROM ITS MEAN OR UNDISTURBED POSITION IS CALLED THE AMPLITUDE OF THE VIBRATION. This would be the distance of a_1 or a_3 from the mean position a_0. So long as the amplitude is not excessive the frequency of the vibrations is independent of the amplitude. You can test this with a pendulum.

There is still another term which we must introduce: PHASE. To designate the phase of a particle in the wave at some instant we must specify what part of the wave, crest, or trough, etc., is passing by the particle. In the diagram the particles at B and F are in the same phase; similarly, the particles at D and H; at A, E, and I; at C and G. (Can you say why A and C do not have the same phase? To answer the question draw a picture of the wave a short time later and see in what directions A and C have moved.)



You must have noticed that the crests and troughs are repeated after a certain distance. This distance is called the WAVE-LENGTH, λ. To be more general we may say that the WAVE-LENGTH IS THE SHORTEST DISTANCE BETWEEN TWO PARTICLES HAVING THE SAME PHASE. In the preceding diagram this will be BF, DH, etc. Can you read off some other distance which is just one wave-length long?§§ Not only does the wave form repeat itself after a distance, λ, but it also repeats itself for any particle in the wave after a time, τ, the period. It is this repetition in space and time that makes wave motion so elusive.

VELOCITY OF A WAVE: A wave travels a distance λ, its wave-length, in a time τ, its period. Hence the velocity of the wave, $v = \left(\frac{s}{t}\right) = \frac{\lambda}{\tau}$. Since $n = 1/\tau$, $v = n\lambda$. This relation, **velocity = frequency × wave-length**, is a very important one in wave motion.

Perhaps an illustration of the use of the equation $v = n\lambda$ may appeal to you. If the frequency of a wave is 100 per sec., 100 waves are passing any point on the wave every second. If the wave-length of each wave is 25 cm., a wave starting out at the beginning of the second will have traveled 100×25 cm., i.e., 2,500 cm., by the end of the second.

$$v = n\lambda = 100 \times 25 = 2{,}500 \text{ cm. per sec.}$$

While the relation $v = n\lambda$ is always true, the actual velocity with which a wave travels through a medium depends on the properties of the medium itself.

There are two cases which are of interest to us here:

a) Velocity of longitudinal waves in a gas:

$$v = \sqrt{\frac{\gamma \times \text{pressure of the gas}}{\text{density of the gas}}}$$

$\gamma = 1.44$ for air and diatomic gases*

$\gamma = 1.66$ for He and other monatomic gases

EXAMPLE. Let us calculate the velocity of longitudinal waves in air at STP:
Pressure of air $= 76 \times 980 \times 13.6 = 1.01 \times 10^6$ dynes per sq.cm.
Density of air $= 0.00129$ gm. per cc.

$$v = \sqrt{\frac{1.41 \times 1.01 \times 10^6}{0.00129}} = \sqrt{\frac{1.41 \times 1.01 \times 10^6}{1.29 \times 10^{-3}}} = \sqrt{1.10 \times 10^9}$$

$$= 3.32 \times 10^4 = 33{,}200 \text{ cm. per sec., or } 332 \text{ m. per sec.}$$

b) The velocity of transverse waves in a rope is given by

$$v = \sqrt{\frac{\text{tension in the rope}}{\text{mass per cm. of the rope}}}$$

In general the velocity of a wave in a medium is given by

$$v = \sqrt{\frac{\text{elasticity of the medium}}{\text{density of the medium}}}$$

We must say something of what we mean by the term *elasticity*. You know that if you deform an object in any way by applying a force to it, when the force is removed the object has a tendency to return to its original shape or volume. This tendency to recover its original

§ Answer to question on the previous page: A quarter of the period $\tau/4$.

§§ Answer to question on the previous page: *AE*; *CG*; *EI*.

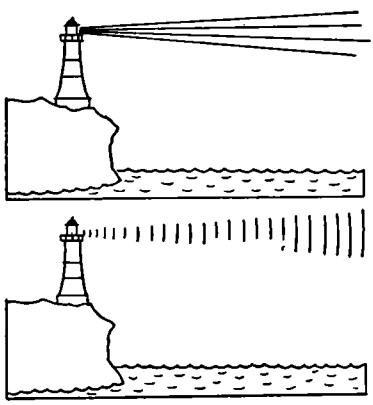
* The factor γ occurs because, when longitudinal waves travel through air, the air is alternately heated and cooled as the compressions and rarefactions come along. Actually, the factor γ is the ratio of the specific heat of a gas at constant pressure to the specific heat at constant volume.

shape or volume is called *elasticity*. In a longitudinal wave the particles of the medium are displaced in the direction of motion of the wave, whereas in a transverse wave the particles are displaced at right angles to this direction. You can see we are running into deep water, so we can merely state that this difference causes a longitudinal wave to travel *faster* through a solid than a transverse wave.

In an earthquake both types of waves are transmitted through the outer portion of the earth. From a knowledge of the velocity of longitudinal and transverse waves through the earth, it is possible to calculate the rigidity of the earth. This turns out to be greater than that of steel!

For a medium to be able to transmit a transverse wave the medium must possess *rigidity*. Now in our discussion of fluids we stated that a fluid has *no* rigidity, so this means that a fluid (liquid or gas) *cannot* transmit a transverse wave through it. Of course at the surface conditions are quite different, for there we are able to get the familiar water waves.

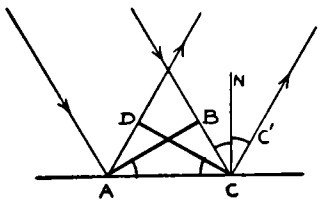
Before considering further the velocity of a wave, we must remind you of something already familiar, the *wave fronts*. When you drop a stone in water you see circular waves, crests, or troughs moving out from the splash. These concentric circles are wave fronts. Actually each circle is the locus of adjacent particles having the same phase. It is not necessary for wave fronts to be circular. They may have almost any shape. One important case is that of the straight and parallel wave front. We can get such straight and parallel wave fronts, called *plane waves*, by going a long way from where the stone is dropped into the water. The circles are then so big that any small portion of them will be nearly straight. Sometimes, purely for convenience, lines are drawn at right angles to the wave fronts to indicate the direction in which the wave fronts are traveling. Such lines are called *rays*.

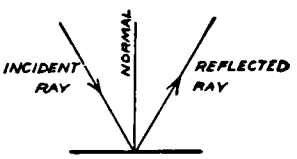


REFLECTION OF WAVES: When a series of waves strike a large smooth surface they are reflected in a manner familiar to you. Thus, when a stone is dropped in a pond and the waves produced strike a large plane surface, the reflected waves appear to have come from a source as far behind the surface as the original disturbance is in front.

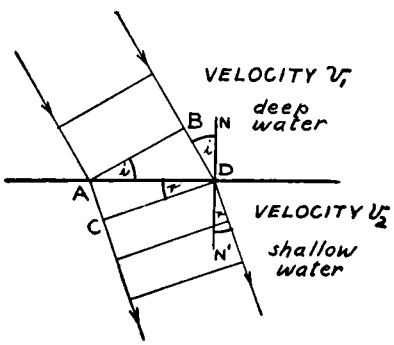
Consider a group of plane waves striking a surface AC. The wave fronts of the incident waves are parallel to AB and those of the reflected rays are parallel to DC. While the wave at B is traveling to BC, the portion at A is reflected and will have traveled back a distance $AD=BC$. Now BC and

CC' are rays, for they are drawn perpendicular to the incident and reflected wave fronts. The line NC is the *normal* at C, i.e., the angle $NCA = 90°$. Consider the triangles ADC, ABC: Since AD is equal to BC, angle ADC = angle $ABC = 90°$; and since AC is common, the triangles are congruent. Therefore angle BAC = angle DCA. So the reflected wave front makes the same angle with the reflecting surface AC as the incident wave front does. The normal NC is perpendicular to AC. The reflected ray CC' is perpendicular to the reflected wave front DC. Similarly, BC, an incident ray, is perpendicular to the incident wave front AB. Then angle BCN = angle $C'CN$. The angle between the incident ray and the normal, angle BCN, is called the *angle of incidence*. Similarly, angle $C'CN$ is called the *angle of reflection*. WHEN A WAVE IS REFLECTED, THE ANGLE OF INCIDENCE IS EQUAL TO THE ANGLE OF REFLECTION. Also, the incident ray, the reflected ray, and the normal all lie in the same plane. This is, of course, the plane of the paper of the diagram.





REFRACTION OF WAVES: Let us now consider what happens when a wave comes to a place where its velocity is changed. The case of water waves moving into *shallower* water where their velocity is *reduced* will do well enough. The frequency of the waves is *unchanged* in going from the deep to the shallower water, but their velocities and wave-lengths are changed. Suppose we have plane water waves of frequency n and wave-length λ_1 moving with a velocity v_1 into shallower water where their wave-length is λ_2 and their velocity v_2. The diagram is drawn looking down onto the surface of the water. The lines AB and CD represent the wave fronts in the water at the different depths.



$$v_1 = n\lambda_1; \qquad v_2 = n\lambda_2; \qquad \text{so that } n = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

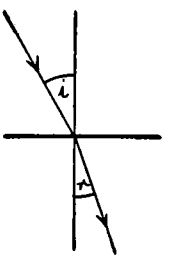
$$\therefore \quad \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

Hence the wave-lengths are reduced in the ratio of the velocities. Let us consider the wave fronts, AB in deep water and CD in the shallower water. In

the time the wave is traveling from B to D in the deep water it goes only a distance from A to C in the shallow water. The ratio of BD to AC is the same as the ratio of the two velocities v_1 and v_2:

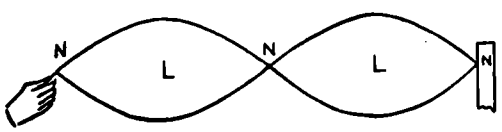
$$\frac{BD}{AC}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}.$$

Since AC is smaller than BD, the wave front must have changed its direction so that the wave now travels more nearly perpendicular to the boundary. The ray has been bent in going from the deep to the shallow water. This *bending* of a ray when it goes into a medium where its *velocity is changed* is called *refraction.* Waves are always bent so that they travel more nearly perpendicular to the boundary when they go from a medium where their velocity is larger to one where their velocity is smaller. What happens when the oncoming waves strike the boundary perpendicularly? Are the wave fronts bent?§



SUPERPOSITION OF TWO WAVES: A very important property of waves is that their velocity is *independent* of the amplitude of the vibrations. This is true except when the amplitude is so very large that the medium is permanently strained by the wave. We shall not deal with such cases. The fact that the velocity of a wave is independent of the amplitude allows a medium to transmit two sets of waves at the same time. The resultant amplitude produced at any point is simply the sum of the separate amplitudes of the two waves.

Let us add two waves together which are traveling in opposite directions. This you can do for yourself by tying one end of a fairly long clothesline to a post and shaking the other end with a regular frequency. If you adjust the frequency of the shakes with your pull on the rope, you find that you can get *standing waves.* The waves are not moving forward as in the previous cases. What you are doing is to superimpose two sets of waves of the same *frequency, velocity,* and *wave-length,* one set traveling *forward* from your hand and the other reflected from the end tied to the post and traveling in the *opposite* direction. When a crest of the forward wave is superimposed on a crest of the reflected wave, a crest of double amplitude results. (It may help you to realize what is occurring to work problem 10 now.)

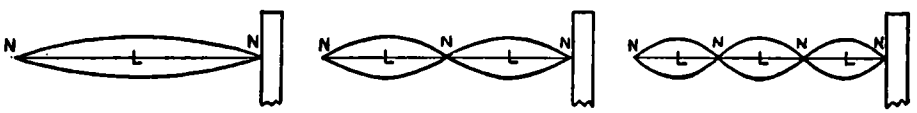


There are some places on the rope which do not vibrate at all. These are

called *nodes* (N). Midway between two nodes are places where the amplitude of vibration is a maximum. These are called *loops* (L). The distance between two nodes or two loops is half a wave-length, $\lambda/2$. Do you see why?

If you try this experiment with the clothesline, you find that by changing the tension you are able to make the rope vibrate with one loop, two loops, three loops, etc. The end tied to the post will be a node and the end where your hand shakes the rope will be approximately a node. The velocity of the forward and reflected waves is given by $v=\sqrt{\frac{\text{tension}}{\text{mass per cm.}}}$ and the wave-length depends on the frequency, n, of the shakes, since $v=n\lambda$. For the standing waves, λ cannot have any value but can only be half the length of the string, the length of the string, etc. With one loop the wave-length is twice that with two loops.

EXAMPLE. Let us try this experiment with a rope 5 m. long. We shall try to pull on the rope with a constant tension of 8,000 gm. wt. If the rope has a total mass of 245 gm., let us find the frequency with which it is necessary to shake the rope to produce a single loop. (We shall, for the sake of simplification, assume that our hand is exactly at a node.)



Velocity of wave in rope: $v=\sqrt{\frac{\text{tension}}{\text{mass per cm.}}}$

Tension $=8{,}000\times 980$ dynes; mass per cm. $=\frac{245}{500}$ gm. per cm.

$$\therefore\quad v=\sqrt{\frac{8{,}000\times 980\times 500}{245}}=4{,}000 \text{ cm. per sec.}$$

But $v=n\lambda$; λ is in this case twice the length of the rope, i.e., 1,000 cm.

$$\therefore \text{ Frequency of shakes}=n=\frac{v}{\lambda}=\frac{4{,}000}{1{,}000}=4 \text{ per sec.}$$

Suppose we now wish to get two loops in the rope. We shall do this by changing the frequency of shaking and not by changing the tension in the rope. The velocity of waves in the rope is still 4,000 cm. per sec. But now λ is equal to the length of the rope (one-half its former value). $\therefore\ n=\frac{4{,}000}{500}=8$ per sec. Thus we must increase the frequency of our shakes to 8 per sec.

We could have produced two loops by keeping the frequency at 4 per sec. but increasing the tension in the rope (by pulling harder). Let us find the tension in the rope which would be necessary to produce two loops if the frequency of shaking is 4 per sec.

§ Answer to question on the previous page: The velocity and wave-length are decreased but there is no bending.

$$\text{Velocity of waves in rope: } v=\sqrt{\frac{T}{m}}=\sqrt{\frac{T\times 500}{245}} \text{ cm. per sec.}$$

$$\text{But } v=n\lambda=4\times 500=2{,}000 \text{ cm. per sec.}$$

$$\therefore \quad \sqrt{\frac{T\times 500}{245}}=2{,}000$$

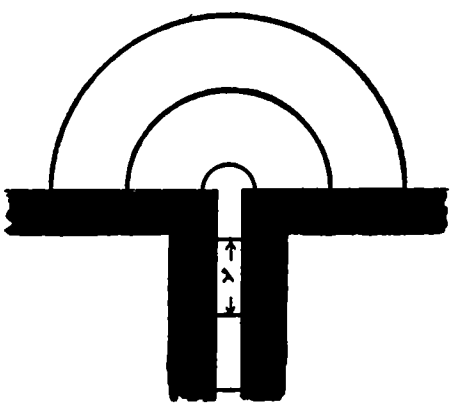
$$\text{or } T=\frac{2{,}000\times 2{,}000\times 245}{500} \text{ dynes}=\frac{2{,}000\times 2{,}000\times 245}{500\times 980}=2{,}000 \text{ gm. wt.}$$

This tension is only one-quarter of that used to produce one loop at the same frequency of shaking, i.e., 4 per sec.

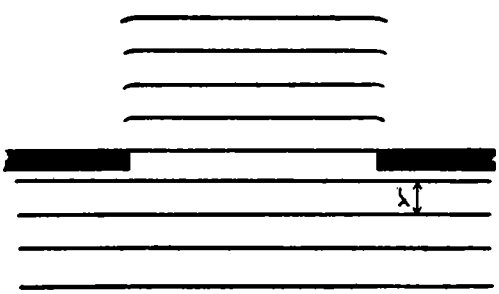
Problem: a) Find the frequency of shaking necessary to produce three loops when the tension in the rope is 8,000 gm. (*Answer:* 12 shakes per sec.)

b) Find the tension in the rope necessary to produce three loops when the frequency of shaking is 4 per sec. (*Answer:* $\frac{8{,}000}{9}$ gm. wt.)

DIFFRACTION: One of the most interesting properties of waves is that of bending round the corners of an obstacle. You must all have seen waves coming out of a narrow opening in a harbor and noticed that they spread out in almost perfect circles when they get into the open sea. It is as though each



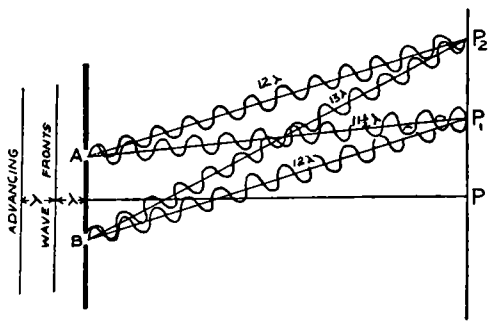
Diffraction of waves of large wave-length going through a narrow opening



Diffraction of waves of short wave-length going through a wide opening

point on the wave front acted as a center of a new disturbance. The amount of spreading of the waves in the open sea depends on the size of the opening and the wave-length of the waves. The *smaller* the opening and the *greater* the wave-length, the *greater* the spreading. A similar phenomenon occurs when water waves hit a projecting rock, the effect again depending on the size of the rock relative to the wave-length of the waves. This bending of the waves round the corner of an obstacle is called *diffraction.* It occurs whenever the wave front is intercepted by some obstacle.

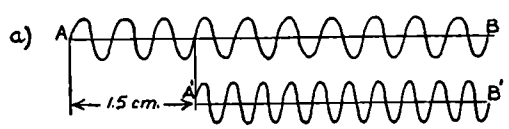
An interesting experiment can be performed with two openings or slits. In this case the diffracted waves from each opening superimpose themselves to give what is called an *interference pattern*. Suppose plane waves are incident on the two slits A and B and we investigate what the resultant waves look like along the line P_1P_2. The waves from A and B start out *in phase*, i.e., if a crest is coming through A, a crest is also coming through B. If the distance $P_1B - P_1A$ is half a wave-length, $\lambda/2$, a crest arriving at P_1 from A will be met by a trough arriving at the same time at P_1 from B. Where a crest and trough are superimposed they neutralize each other. If we move to a place P_2 such that $P_2B - P_2A$ is a whole wave-length, λ, a crest arriving at P_2 from A will be met by a crest arriving at the same time from B. Do these two crests arriving at P_2 at the same time start out from A and B at the same time? If not, what is the time difference?§



Thus if $P_1B - P_1A = \lambda/2$, a wave of zero amplitude results;
and if $P_2B - P_2A = \lambda$, a wave of double amplitude results.

Moving still farther out to a point P_3 such that $P_3B - P_3A = 3\lambda/2$, there will again be a place of zero amplitude. And so on. This interference pattern will be repeated on the portion below P_2P_1.

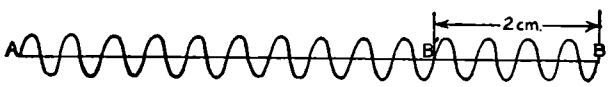
DOPPLER EFFECT: Let us now consider the case of waves sent out by a source which is in motion. Suppose the source sends out 10 waves per sec. and that the wave-length of these waves is 0.5 cm. We wish to see what happens to these waves if the source moves with a velocity of 1.5 cm. per sec.



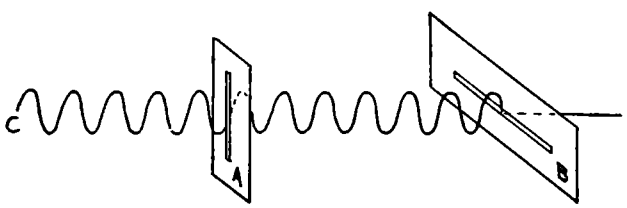
In diagram *a*, where the source at A is at rest, the first of the 10 waves has traveled to B, a distance of 5 cm. during the 1 sec. During this second the source has moved a distance of 1.5 cm. to A', so that the 10 waves are now contained in a distance of 3.5 cm.; their wave-length is 0.35 cm. An observer at B' will receive waves of *shorter wave-length when the source approaches him* than when the source is at rest. Since the waves now have a shorter wave-length, they must have a correspondingly *greater frequency.*

Their frequency was 10 per sec. when A was at rest and their velocity, $v = n\lambda = 10 \times 0.5 = 5$ cm. per sec. The velocity of the waves when A moves is still 5 cm. per sec., but their wave-length is 0.35 cm. Hence their frequency under these conditions must be $v/\lambda = 5/0.35 = 14.3$ vibrations per sec. If the source moves away from the observer, what is the effect on the wave-length and frequency?§

Another interesting situation exists when the source is at rest but the observer moves. In the case when the observer was at rest, 10 waves per sec. passed him; but now if he moves toward the source—say a distance of 2 cm. in a second—4 *extra* waves per sec. will pass him. In all, then, 14 waves per sec. pass him when he moves toward the source with a velocity of 2 cm. per sec. How many waves per sec. pass him when he moves away from the source with a velocity of 2 cm. per sec.?§§ This effect of change in *wave-length* or *frequency* due to the motion of the source or of the observer is called the DOPPLER EFFECT.



All the phenomena we have described so far are exhibited by any type of wave. The question arises as to whether there is any way in which we may differentiate between a longitudinal and a transverse wave. Of course if the waves are visible, we might be able to tell by looking at them; but many waves are not visible. Let us consider a transverse wave in a rope passing through a long but relatively narrow slit cut in cardboard. If the slit is placed in position A, the wave will pass through unimpeded; but if a second slit is placed as shown in position B, the wave will be stopped. The wave in the portion CA is a *plane polarized* wave since the vibrations take place in a single plane. If a whole series of waves with their planes of vibration in every direction are sent through A, only the one with its plane parallel to the slit will get through unimpeded. This would be eliminated by slit B. By using two slits with their axes at right angles, it is possible to eliminate a transverse wave, but no such thing can be done with a longitudinal wave. The vibrations of the particles of the medium are in the same direction as the wave is traveling, so that if the wave gets through one slit it can get through another whatever the orientation of the two slits.



§ Answer to question on the previous page: No. The period of the wave.

PROBLEMS

1. A stone dropped into a pond produces waves which travel outward with a velocity of 125 cm. per sec.
   a) Find how far the waves travel in 10 sec.
   b) If the wave-length of the waves is 100 cm., what is their frequency?

2. The elasticity of steel is 1.8×10^{10} dynes per sq.cm. and its density is 7.8 gm. per cc.
   a) Find the velocity of longitudinal waves in steel.
   If the frequency of the waves sent through the steel is 1,000 per sec.,
   b) What is the wave-length of these waves?

3. Longitudinal waves are sent through two mediums, A and B, each of which has the same density. In medium A the force pulling a displaced particle back to its undisturbed position is greater than in B. In which medium, A or B, would you expect the waves to travel faster?

4. A disturbance which gives rise to an earthquake occurs some distance below the surface of the earth.
   a) Which type of wave, longitudinal or transverse, would first affect a distant seismograph (an instrument for detecting earthquake waves)?
   b) If the central core of the earth is liquid, will both types of waves be transmitted through this core? If not, which one will be stopped?

5. Waves from earthquakes or violent explosions not only travel through the earth but also along the surface of the earth. Waves which travel along the surface are very complex, but in general we may say that they travel about 3,000 m. per sec. This is about half as fast as the transverse waves travel through the earth.
   a) Suppose an explosion takes place 5,000 m. away from where you are. This explosion produces waves which travel along the surface of the earth and through the air. Which ones reach you first?
   b) What would be the interval of time between the arrival of the two types of waves? (Take the velocity of sound in air as 33,000 cm. per sec.)

6. Enemy operations underground were detected during the war by means of apparatus which recorded waves traveling along the surface of the ground at several stations.

   Suppose three detecting stations, A, B, and C, are placed in a straight

§ Answer to question on the previous page: The wave-length is increased to 0.65 cm. and the frequency is decreased to $\frac{5}{0.65} = 7.7$ vibrations per sec.

§§ Answer to question on the previous page: 6.

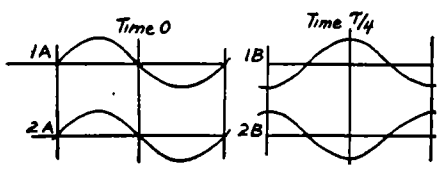
line and that the distance from A to B is equal to the distance from B to C, i.e., 300 m. On a certain day waves are received at A and C at the same instant, but at B they arrive $\frac{1}{30}$ sec. earlier. Find the distance and location of the source of the waves from A, B, and C.

7. *a*) Can you say why waves coming toward a shelving shore bend round so that their wave fronts follow the shape of the coast line?
   b) Of what is this an example?
   c) If the coast were made up of cliffs so that the water was very deep at the foot of the cliffs, would the waves have the same tendency to conform to the shape of the coast line?

8. When waves are breaking against a cliff, would you expect them to do more damage if they loosen bowlders and hurl these against the cliffs than if there were no bowlders? (Consider how the energy of the waves is dissipated if the waves are free of bowlders.)

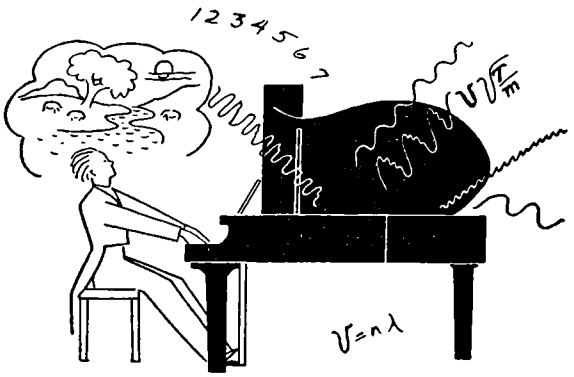
9. You are standing on a bridge quietly watching the waves sent out at the bow of a boat coming toward you down the river.
   a) Do you receive more, less, or the same number of waves per sec. when the boat is coming toward you as when the boat has passed under the bridge and is moving away from you?

10. Graphs $1A$ and $1B$ represent transverse wave forms. Graph $1B$ is the wave form a quarter of a period, $\tau/4$, later than Graph $1A$. Graph $2B$ is a quarter of a period later than graph $2A$.
   a) In which direction, to the left or to the right, is the wave shown in Diagram 1 moving?
   b) In which direction is wave 2 moving?



We have shown that standing waves are produced by the superposition of two similar waves traveling in opposite directions. Copy the graphs and then draw to the right of them corresponding pictures of the waves at times $2\tau/4$ and $3\tau/4$. Form a third graph vertically underneath these by adding the amplitudes of $1A$, $2A$, etc. These give the picture of the standing waves produced at times 0, $\tau/4$, $2\tau/4$, $3\tau/4$.

11. If the period of the waves in problem 10 is 50 per sec. and the crest moves a distance of 1 cm. in the quarter-period,
   a) What is the wave-length of these waves?
   b) What is the velocity of the waves shown in Diagram 1?

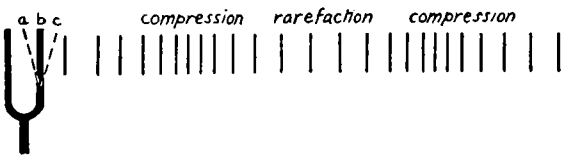


CHAPTER 13

SOUND

Sound travelling far and wide
A stormy day will betide.
—WEATHER PROVERB

YOU ARE all familiar with the multitude of ways of producing sound: violins, organ pipes, tuning forks, vibrating disks, etc. All these instruments set the air into regular vibrations which are transmitted as longitudinal waves to the ear, where they produce the sensation of sound. We shall use a tuning fork as our vibrating source. When the prongs are struck, they vibrate to and fro, alternately compressing the air in front (as they move from *a* to *c*) and rarefying it (as they move from *c* to *a*). These *compressions* and *rarefactions* are transmitted through the air in the form of longitudinal waves. We should expect from the way in which the waves are produced that they would be longitudinal and not transverse. This is supported by the fact that sound waves cannot be polarized.



Since each complete vibration of the prongs of the tuning fork sends forth a compression and a rarefaction, the frequency of the tuning fork must also be the frequency of the waves sent out. The PITCH of the note is determined by the FREQUENCY of the waves. The greater the frequency, the higher the pitch. If the tuning fork is struck a hard blow, we know that a relatively loud sound is received by the ear. The hard blow makes the amplitude of the vibrations of the prongs of the fork large and hence the amplitude of the waves large. The amplitude of the sound waves thus determines the loudness of the sound.

WE HAVE already calculated the velocity of sound waves in air at STP and found it to be 332 m. per sec.* You know now all that is necessary for

* See p. 146.

calculating the effect of change of pressure and temperature on the velocity of sound waves, so try problems 6 and 7 before proceeding.

If you have sufficiently recovered from these problems, you now know that *the velocity of sound is not affected by a change in pressure if the temperature is constant,* but that *the velocity increases as the square root of the absolute temperature if the temperature changes while the pressure remains constant.*

An interesting and very important property of sound waves is that *waves of all frequencies travel with the same velocity.* A high-pitched note, whether loud or soft, travels with the same velocity as a low-pitched note. Just think of the hopeless jumble of noise you would get from an orchestra if this were not the case.

SOUND WAVES are reflected. The most familiar example of this is an echo. Doesn't everybody like to sing in the bath where bare walls reflect the sound back and forth? In a drawing-room, where there are cushions and carpets, the sound is absorbed to a great extent, so that there is little reverberation.

The human ear is limited in the range of frequencies of longitudinal waves which it can hear. Waves of frequencies below 20 per sec. are too low in pitch to be heard, while those above about 20,000 per sec. are too high in pitch to be audible. The audible range varies somewhat with different persons.

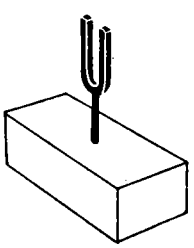
EXAMPLE. Let us find the wave-length of the sound waves emitted at 0° C. by a tuning fork having a frequency of 200 per sec.

$$v = n\lambda \qquad v = 33{,}200 \text{ cm. per sec.}; \qquad n = 200 \text{ per sec.}$$

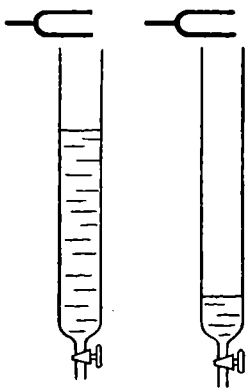
$$\lambda = \frac{v}{n} = \frac{33{,}200}{200} = 166 \text{ cm.}$$

The wave-lengths of audible sound are much longer than those of visible light, so we should expect from our previous discussion on *diffraction* that there would be considerable bending of sound waves round a corner of an obstacle. You know from experience that sound waves bend round corners; you can hear people talking in the hall before you see them pass the door.

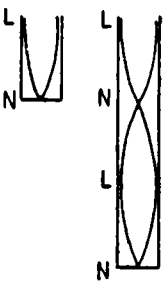
STANDING WAVES AND RESONANCE: Tuning forks are usually mounted on sounding boxes open at one end. If you remove the fork from the box the loudness of the sound is considerably decreased. The hollow sounding box has to be of a suitable length for the particular fork, otherwise the effect is lost. We shall investigate this phenomenon by holding a vibrating tuning fork over a column of air whose length can



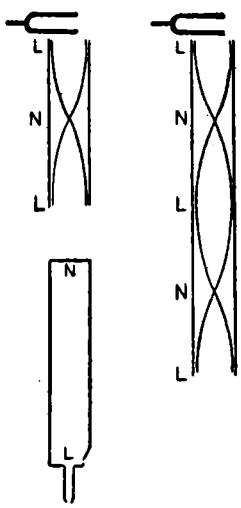
be changed by changing the level of water in a tube. When the water stands at the positions shown in the diagrams, the sound increases considerably in loudness. The column of air at these positions is *resonating*, i.e., it is set into sympathetic vibration with the fork. The fork is sending out compressions and rarefactions down the air column. The frequency of the fork and the length of the air column must be related in such a manner that, during the time taken by a compression or rarefaction to go down the tube and back, the fork makes half a complete vibration. The compressions and rarefactions from the fork and the air column reinforce one another.



It is perhaps easier to consider this phenomenon from the point of view of the standing waves set up in the air column. The air just in contact with the water cannot vibrate. Hence there is a *node* there. At the top of the tube the air is free to make its maximum vibration so a *loop* exists there. Since the distance from a node to the next loop is a quarter of a wave-length, $\lambda/4$, this must be the length of the shorter air column, while the length of the longer air column must be $3\lambda/4$.



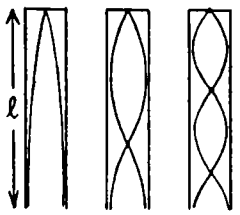
The wave-length, λ, is determined from the frequency of the fork. Thus in an earlier example we found that $\lambda = 166$ cm. when the frequency was 200 per sec. The length of the shorter column of air in resonance with this fork would be $\frac{166}{4}$, i.e., 41.5 cm. The next longer column of air which would be in resonance would be $\frac{3\times 166}{4}$, i.e., 124.5 cm. Can you say what would be the next length of air column which would resonate with this fork?§



Suppose we hold the vibrating fork over a tube which is open at both ends and whose length can be changed. For resonance to take place each end of the tube must be a loop. The shortest length of tube resonating to the fork of frequency 200 per sec. is $\lambda/2$, i.e., 83 cm. A tube twice as long as this would also resonate to the same fork.

Let us now consider an organ pipe closed at the upper end. Wind is blown in through a tube at the lower end and a lip or reed near by causes vibrations of a large number of frequencies to be sent out. The pipe selects out of these frequencies those which resonate with the pipe. It is these notes which are heard. The end near the lip must be

a loop of motion, for there the air is free to vibrate. The closed end must be a node. Three possible modes of vibration are shown in the diagram.



EXAMPLE. Suppose the length of the organ pipe is 1 m. Let us find what notes will be sent out from it.

a) The wave-length λ_1 of the note resonating is four times the length of the tube, i.e., $4\times100=400$ cm. The frequency of this note is n, where $v=n\lambda$.

$$n=\frac{v}{\lambda}=\frac{33,200}{400}=83 \text{ vibrations per sec.}$$

This note is the FUNDAMENTAL one and is the one characteristic of the pipe.

b) In this case the wave-length λ_2 is equal to $\frac{4}{3}l=\frac{400}{3}=133.3$ cm. This is one-third of the wave-length of the fundamental. The frequency of this note, n_2, is equal to

$$\frac{v}{\lambda}=\frac{33,200\times3}{400}=249 \text{ vibrations per sec.}$$

This note is called the FIRST OVERTONE of the organ pipe and its frequency is three times that of the fundamental. It is not so loud as the fundamental.

c) Wave-length: $\lambda_3=\frac{4}{5}\times l=\frac{400}{5}=80$ cm. This is one-fifth the wave-length of the fundamental.

$$\text{Frequency of note: } n_3=\frac{v}{\lambda_3}=\frac{33,200}{80}=415 \text{ vibrations per sec.}$$

This frequency is five times that of the fundamental. This is the SECOND OVERTONE of the organ pipe. Can you suggest what the frequency of the next overtone will be?§

Coming from this organ pipe there will be a composite note, the fundamental with its overtones. The tone we hear will be different from the fundamental alone. It will have a richer QUALITY. The QUALITY of a musical note is determined by the intensity and frequency of the overtones accompanying the fundamental. If the fundamental has a frequency of n per sec., the frequencies $2n$, $3n$, $4n$, etc., are called the HARMONICS of this frequency n. In a closed organ pipe the overtones consist of the odd harmonics, i.e., $3n$, $5n$, $7n$, etc.

YOU KNOW that certain notes sounded together on a piano form a pleasing chord, while others taken at random produce a discord. This discord apparently arises from interference between the overtones of the notes sounded. BEAT NOTES are heard. Suppose we sound together two tuning forks which have frequencies of 200 and 202 vibrations per sec. What we hear is a note which rises and falls in loudness twice per sec. Twice each second the waves of the notes are *in phase*, i.e., they are producing

§ Answer to question on the previous page: $\frac{5\times166}{4}=207.5$ cm.

condensations or rarefactions together. This note having the frequency of 2 per sec. is called a BEAT NOTE and its frequency is the difference in frequency of the two notes sounded. Heterodyning in radio uses this principle of beats. The frequency of the incoming radio waves is very high. It is combined with a constant high frequency produced in the radio set. The two frequencies produce beats—the beats in this case having the frequency of audible sound.

PROBLEMS

1. If you hear the thunder 5 sec. after you see the lightning, how far in centimeters is the storm from you? What is the approximate distance in miles? (The velocity of light is so great that the time taken for the lightning to reach you is negligible.) (1 mi. = 5,280 ft.; 1 ft. = 30 cm. approx. Take the velocity of sound as 330 m. per sec.)

2. An echo from a mountain side is heard 1 sec. after you shout. How far away in meters is the mountain?

3. Calculate the velocity of sound in hydrogen at STP from the formula

$$v = \sqrt{\frac{1.41 \times \text{pressure}}{\text{density}}}$$

Standard pressure = 76 cm. of mercury = 10^6 dynes per sq. cm.
Density of hydrogen at STP = 8.9×10^{-5} gm. per cc.

4. What is the ratio of the velocity of sound in oxygen gas to that in nitrogen gas at the same temperature and pressure? (Use molecular weights and Avogadro's hypothesis. Leave your answer as a square root.)

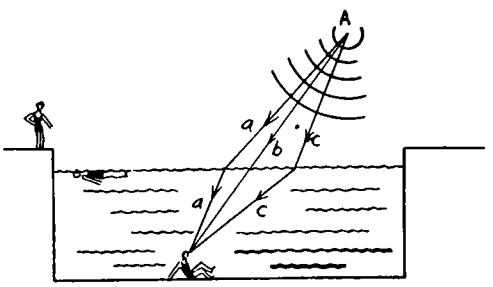
5. What is the ratio of the velocity of sound in helium gas to that in hydrogen gas at the same temperature and pressure? (Leave your answer as a square root.)

6. Find the effect on the velocity of sound in dry air of an increase in pressure when the temperature is constant. The following questions may help you to arrive at your answer:
   a) When the temperature of the air is constant, how is its density affected by an increase in pressure?
   b) What law do you use?
   c) Hence what is the effect on the velocity of sound of a change in pressure at constant temperature?
   d) Does this argument apply to any gas?

§ Answer to question on the previous page: 581 vibrations per sec.

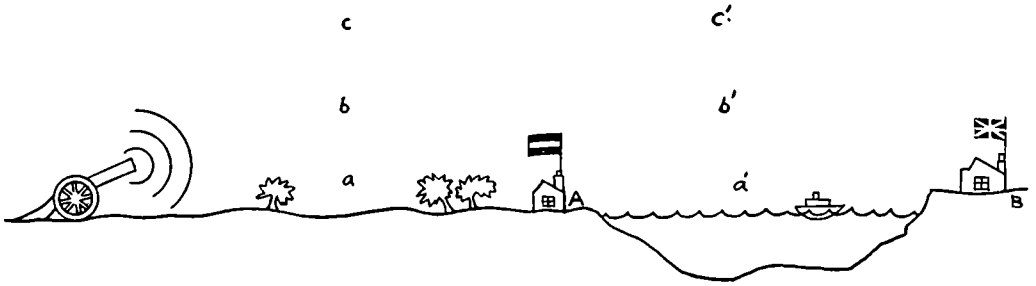
7. Find the effect on the velocity of sound in dry air due to an increase in temperature when the pressure is constant. Use the following questions to arrive at your answer:
   a) If the density of air at 0° C. (273° A.) is d_0 and its density at a temperature of 27° C. (300° A.) is d_{27}, what is the value of the ratio d_0/d_{27}?
   b) What law are you using?
   c) If d_0 is the density of air at 0° C. (273° A.) and d_t is its density at T° A. (273+t° C.), what is the value of the ratio d_0/d_t?
   d) If the velocity of sound at STP is v_0 and its velocity at T° A. and the same pressure is v_T, what is the value of the ratio v_T/v_0?

8. *a*) Do longitudinal waves travel faster in air or in water? A person sitting temporarily on the floor of a swimming pool hears sound coming from a hooter at *A*.
   b) Which path (*a*, *b*, or *c*) do the sound waves take to reach him?



9. In the case of a stringed musical instrument such as a violin, standing waves are set up in the vibrating catgut strings.
   a) Are these standing waves transverse or longitudinal?
   b) Can these standing waves produce longitudinal waves in the air?

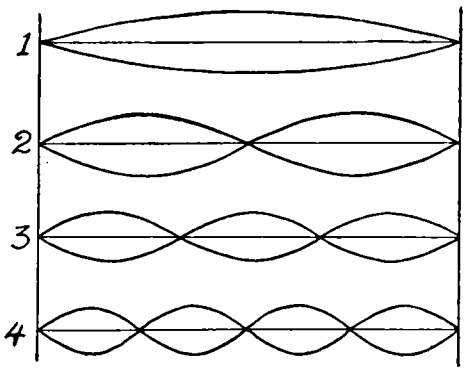
10. During the war it was often noticed that the noise of the guns could be



heard on the coast of England, while at closer places in France they could not be heard. These "noiseless zones" are produced by refraction of the sound waves by wind.
If the guns are not heard at *A* while they are heard at *B*, where is the wind velocity greatest: (1) at *a*, *b*, or *c*? (2) at *a'*, *b'*, or *c'*?

11. *a*) Find the velocity of the transverse waves in a violin string whose length is 100 cm. and whose mass is 1.25 gm., when the tension in the string is 5×10^6 dynes.

b) What is the wave-length of the standing transverse waves when the string is vibrating with one loop? two loops? three loops? four loops?



c) Find the frequency of the waves in each of these cases.

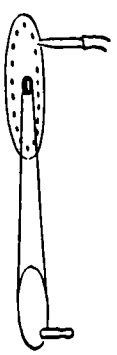
d) Will the frequency of the sound waves emitted be the same as the frequency of the vibrating string in each case?

12. An open-end organ pipe has a length of 1 m.
    a) What is the frequency of the fundamental? (Take the velocity of sound as 332 m. per sec.)
    b) Find the frequency of the first three overtones.
    c) Do the overtones consist of the odd, even, or all the harmonics?

13. *a*) What is the length of an open organ pipe which emits the same fundamental note as a closed pipe whose length is 50 cm.?
    b) Will the quality of the notes given out from both pipes be the same?

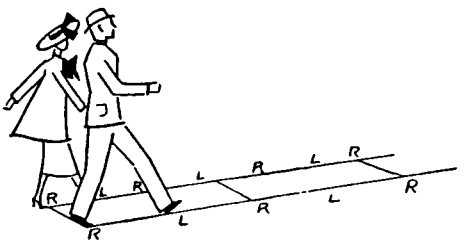
14. Two farmers whose fields are 359 m. apart communicate with each other by shouting one hot summer's day when the temperature is 45.5° C. and the pressure 78 cm. of mercury. The average frequency of their voices is 200 per sec.
    a) Find the velocity of sound on this day. $\left(\text{Take } \sqrt{\frac{318.5}{273}} = 1.08.\right)$
    b) Find how long it takes one farmer's voice to carry to the other.
    The wives then come out and shout at each other in high-pitched voices, the average frequency of which is 1,000 per sec.
    c) Find how long it takes for the word to go from one farmer's wife to the other.

15. In the diagram we see a disk with holes evenly spaced along its outer edge. Notes of definite pitch are produced by turning this disk at a uniform velocity in front of a jet of air. Find the frequency of the note emitted if there are 50 holes in the disk and if the disk makes 3 complete revolutions per sec. The distance from the axis of revolution to the holes is 10 cm.



16. A cheer-leader uses a megaphone. Why does he use it? Contrast the size of his mouth with the wave-length of the sounds he sends forth.
    a) Will the waves from his mouth be diffracted very considerably?
    b) Will the sound waves be diffracted as much from the open end of the megaphone as from his mouth?

17. We may illustrate the principle of beats in sound by a lady and gentleman out for a walk together. The lady takes dainty steps of 18 in. while the gentleman strides at 27 in. per step.
    a) If the velocity of the lady is 3 ft. per sec. (2 MPH), what is the gentleman's velocity?
    b) How many complete steps per sec. does the lady take? the gentleman? (Two steps are necessary for one complete step, i.e., one with the left foot and one with the right.)



    c) If they commence the walk by both starting off with the left foot, how many complete steps does the lady take before they are again both stepping off with the left foot at the same time? how many complete steps has the gentleman taken before this happens?
    d) How many times per sec. are their steps in phase, i.e., are they both stepping off with the same foot, either right or left?
    e) Does this check the formula that the number of beats per sec. produced by two notes of frequencies n_1, n_2 is $n_1 - n_2$?

18. Two exactly similar tuning forks (each having a frequency of 300 per sec.) are mounted on sounding boxes standing near each other.
    a) If one of them is started vibrating and then stopped after a short time, would you expect to find the other vibrating?

    A small piece of wax is placed on the prongs of one of them, causing its frequency to be reduced to 297 per sec.
    b) If the 300 frequency fork is set vibrating and then stopped, would you expect to find the 297 frequency fork vibrating?
    c) If both forks are now set vibrating together, how many beats per sec. will you hear?

19. In Lemon, *From Galileo to Cosmic Rays*, page 367, an experiment is described in which standing waves are produced by the interference of the outgoing waves from a high-pitched whistle with those returning by re-

flection from a board properly adjusted. The positions of the nodes and loops are detected by a sensitive flame. In such an experiment the board was moved back and forth with respect to the flame until a position was found where the flame roared. The board was then moved slowly farther away from the flame. It was found that at a position 2 cm. farther away from the spot where the flame roared the flame was quiet and that 2 cm. (still farther) away the flame again roared.

a) Find the wave-length of the standing waves.

b) Find the frequency of the whistle if the velocity of sound is 332 m. per sec.

c) Would you be able to hear the note from this whistle?

20. A fire truck sounding a siren whose frequency is 1,000 per sec. is coming toward you with a velocity of 1,200 cm. per sec. (27 MPH).

a) Will the frequency you hear be greater than, equal to, or less than 1,000 per sec.?

b) After the truck passes will you hear a note of frequency greater than, equal to, or less than 1,000 per sec.?

21. To find the frequency of the note you hear as the truck of problem 20 comes toward you consider the following:

The 1,000 waves given out per sec. when the truck is at rest are contained in a distance of 33,200 cm. If during the second the truck moves toward you a distance of 1,200 cm., find

a) The distance in which 1,000 waves are now contained.

b) The wave-length of the waves.

These waves travel with the velocity of sound, 33,200 cm. per sec.

c) Find their frequency.

d) Find the frequency of the note you hear when the truck is going away from you. (Follow the same line of reasoning.)

22. A factory whistle gives out a note whose frequency is 1,500 per sec. You are motoring directly toward the factory with a velocity of 3,320 cm. per sec. (73 MPH).

a) Will the frequency you hear be less than, greater than, or equal to 1,500 per sec.?

b) After you pass the factory and move away from it with the same velocity as you approached it, will the frequency you hear be less than, greater than, or equal to 1,500 per sec.?

23. To find the frequency of the note you hear as you approach the factory consider the following: The 1,500 waves given out in 1 sec. are contained in a distance of 33,200 cm.

a) During 1 sec. how far do you move toward the whistle?

b) How many waves are contained in this distance?

c) Hence, what is the total number of waves you receive during this second? What is the frequency of the note you hear?

d) Find the frequency of the note you hear after you pass the factory and are moving away from it with a velocity of 3,320 cm. per sec.?

Emission spectrum of laboratory source
Absorption and emission lines of comet
Emission spectrum of laboratory source

Doppler effect in spectrum of comet

24. See if you can now develop the general formula for the Doppler principle.

You are moving toward a source of waves with a velocity of v cm. per sec. The frequency of the wave is n and its wave-length λ. The velocity of the wave is V cm. per sec. The frequency of the waves reaching you is n'.

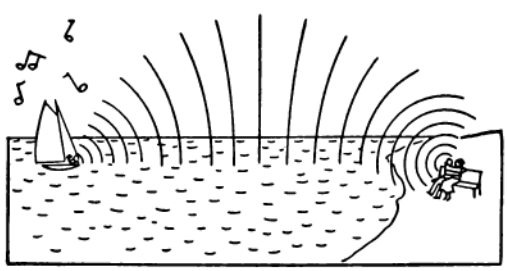
a) Find the ratio n'/n in terms of V and v.

You are at rest but the source of waves is moving toward you with a velocity v cm. per sec. If n' is the frequency of the waves reaching you, and λ' their wave-length,

b) Find the ratio λ'/λ in terms of V and v.

c) Find the ratio n'/n in terms of V and v.

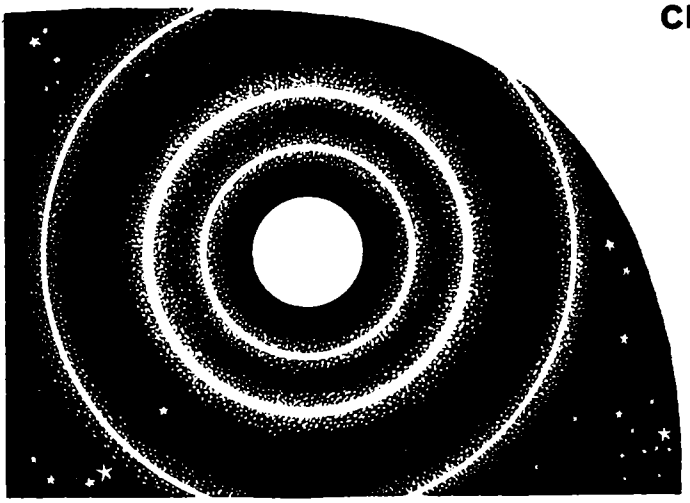
25. You have probably noticed when you have been camping in the country that it is possible to hear people talking very clearly early in the morning before the sun has risen very high. Can you explain this?



a) Is the temperature of the air near the ground cooler or warmer than the air higher up?

b) What effect does this temperature difference have on any sound waves produced near the ground?

CHAPTER 14



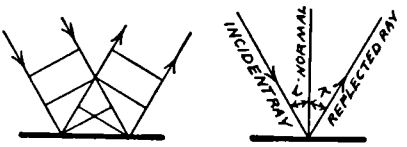
LIGHT

For I fear a hurricane;
Last night the moon had a golden ring,
And tonight no moon we see.
—LONGFELLOW

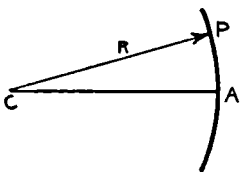
WE ARE going to proceed on the basis that light is transmitted by waves. The proof of this consists in showing that light exhibits DIFFRACTION and INTERFERENCE. Light, unlike sound, etc., requires no material medium to transmit it. Since waves in general need some medium to transmit them, it was felt that light should need one too. Accordingly, a hypothetical medium, called the "ether," was postulated. A considerable advance was made toward the understanding of light when Clerk Maxwell, starting from Faraday's conception of lines of magnetic and electric force, built up a mathematical electromagnetic theory of light. Light was considered by him as the vibrations of electric and magnetic fields at right angles to each other.

FROM THIS theory came a method of determining the VELOCITY of light from purely electrical data. The ratio of the unit of charge in the electromagnetic system (10 coulombs) to the unit of charge in the electrostatic system is numerically equal to the velocity of light, 3×10^{10} cm. per sec. This velocity was first measured by Roemer by carefully noting the eclipses of the moons of the planet Jupiter, and later by Bradley by observations of the stars at the zenith. These astronomical methods were followed in the nineteenth century by the purely physical methods of Foucault, Fizeau, Newcomb, and Michelson. These methods also showed that light travels *slower* in water, glass, etc., than in air.

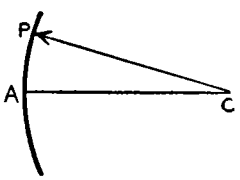
NOW ORDINARILY visible light has a wave-length which is exceedingly small, of the order of 10^{-5} cm., (0.00001 cm.), so we might expect that in order to reflect light waves the reflecting surface must be very smooth. If this is not the case, a wave front incident on the mirror will be scattered in all directions and no regular reflection will take place. We showed in our discussion of waves that they are reflected so that the angle of incidence equals the angle of reflection. Let us digress a little and consider what happens when light is incident on a curved mirror.



CURVED MIRRORS: The mirrors we shall deal with all have a spherical curvature just as though they were cut from a sphere. The two types of mirrors so obtained are called CONCAVE and CONVEX mirrors (see diagram). The reflecting surface of these mirrors is toward the left of the page.



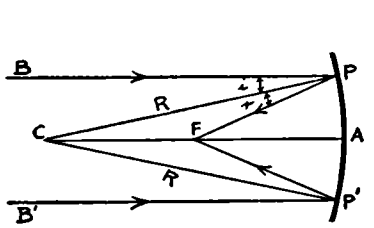
Concave mirror

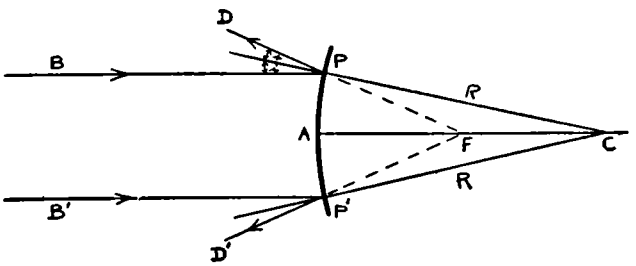


Convex mirror

The *normal* to any point P on the mirror will be the line joining the point P to the *center of curvature* C, i.e., the radius to the point P. The *axis* of the mirror is the line CA. If a light is placed at the center of curvature of the concave mirror, all the rays of light will strike the mirror normally, i.e., they will have an angle of incidence of zero and hence will all be reflected back to the center of curvature. Let us consider what happens when a beam of light whose rays are parallel to the axis strikes the mirror. For simplicity of drawing we shall consider any two rays, BP and $B'P'$, one on each side of the axis.

We shall start with the CONCAVE mirror: The angle of incidence i is the





angle between the incident ray and its normal, $\widehat{BPC}$. The angle of reflection r is the angle between the reflected ray and its normal, $\widehat{CPF}$. By the law of

reflection, $\hat{i}=\hat{r}$. From your geometry you know that since BP is parallel to CA

$$\hat{i}=\widehat{PCF} \qquad \therefore \quad \hat{r}=\widehat{PCF}$$

so that the triangle CPF is an isosceles triangle. Hence

$$CF=FP$$

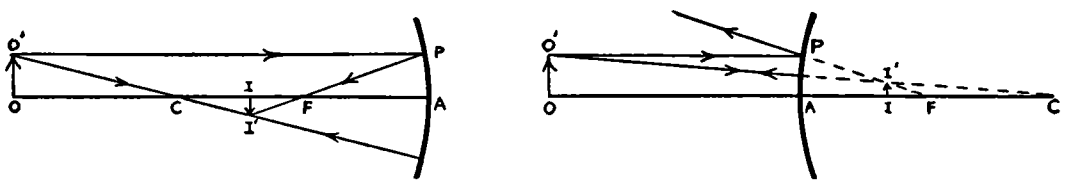
If, *and only if*, P is close to A we may say $FP=FA$.

$$\therefore \quad CF=FA$$

and F is the midpoint of CA. Thus a beam of rays parallel and close to the axis will be reflected to a point on the axis called the *focus* F of the mirror. When the rays are not close to the axis, then after reflection they do not pass through the focus and *aberration* results. You see an example of aberration when a distant light is reflected in a full cup of coffee—the coffee merely serves to show up the reflected rays. We shall limit ourselves to the case of rays close to the axis.

You might prove for yourselves that the same relationship holds for the CONVEX mirror. Of course in this case the rays diverge along PD, $P'D'$, and *only appear* to come from F so that the focus for the convex mirror is a *virtual* focus.

We are now in a position to find the location of the image of any object placed in front of a mirror. To do this we shall use an arrow OO' as the object and place the tail of the arrow on the axis. This is purely for simplicity in drawing. Of all the rays of light which go from the object to the mirror we



shall need only two to determine the position of the image. We need only discuss in detail the CONCAVE mirror because the convex is similar except that the focus and the image are both virtual: One ray $O'P$, from the head of the object O' and parallel to the axis, will be reflected along PF through the focus. A second ray $O'C$, passing through the center of curvature and hitting the mirror normally, will be reflected back along its own path. The head of the arrow I' will be at the point where these two rays meet. The image II' will be obtained by drawing a line from I' to the axis, parallel to the object. To describe the image we must say whether it is *real* or *virtual*,

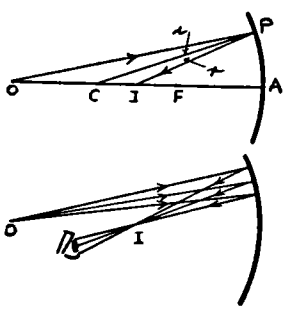
upright or *inverted, magnified* or *diminished.* An image in front of the mirror will be real, while one behind will be virtual. Thus the image II' in the concave mirror is real, inverted, and diminished, while the image II' in the convex mirror is virtual, upright, and diminished.

It is often convenient to find the position, etc., of the image by the use of the formula*

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f},$$

where p = distance of object from mirror, q = distance of image from mirror, f = focal length of mirror, i.e., distance of focus from mirror. This is an algebraic equation where p, q, and f may be positive or negative. A rule which

* If you are interested in the proof of this equation, here it is for the concave mirror. We shall use a diagram similar to the previous one, except that we shall use a point source of light as the object at O. The image at I will be a point too. If you enjoy doing algebraic problems and solving similar triangle problems, you can obtain exactly the same equation by using the previous diagram.



Since I is the image of the object O and CP is the normal for the incident ray OP, the angle of incidence OPC is equal to the angle of reflection IPC. Therefore CP is the bisector of the angle OPI. Now there is a proposition in geometry about the bisector of angles of a triangle which states that

$$\frac{OC}{IC}=\frac{OP}{IP}$$

The ray OP is assumed to be near the axis, so we may set

$$OP=OA=p$$

$$IP=IA=q$$

$$OC=OA-CA=p-R$$

$$IC=CA-IA=R-q$$

$$\therefore\ \frac{OC}{IC}=\frac{OP}{IP} \quad \text{becomes} \quad \frac{p-R}{R-q}=\frac{p}{q}$$

$$\therefore\ pq-qR=pR-pq \quad \text{or} \quad 2pq=qR+pR$$

Dividing all the terms of the equation by pqR, we obtain

$$\frac{2pq}{pqR}=\frac{qR}{pqR}+\frac{pR}{pqR} \quad \text{or} \quad \frac{2}{R}=\frac{1}{p}+\frac{1}{q}$$

$$\text{But } FA=\frac{1}{2}\,CA \quad \text{or} \quad f=\frac{R}{2}, \quad \text{so that} \quad \frac{2}{R}=\frac{1}{f}$$

$$\text{Hence } \frac{1}{p}+\frac{1}{q}=\frac{2}{R}=\frac{1}{f}$$

(If you should try to prove this equation for the convex mirror, remember to put $CA=-R$ and $IA=-q$.)

You can prove that $\frac{\text{Size of image}}{\text{Size of object}}=\frac{q}{p}$ from similar triangles in the previous drawings.

takes care of both the concave and the convex mirrors is: The quantities p, q, and f are positive when in front of the mirror and negative when behind the mirror. Thus the focal length of a convex mirror is a negative quantity. There is also a formula dealing with the relative size of image and object:

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{II'}{OO'} = \frac{\text{image distance } q}{\text{object distance } p}$$

EXAMPLE 1. Let us use the formulas for calculating the position and size of the image given by the CONCAVE mirror on page 168.

$$p = OA = +5 \text{ cm.}; \qquad f = FA = +\tfrac{3}{2} \text{ cm.}; \qquad q = IA = ?$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \text{or} \qquad \frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{2}{3}-\frac{1}{5}=\frac{10-3}{15}=\frac{7}{15}$$

$$\therefore \quad q = +\frac{15}{7} = 2\tfrac{1}{7} \text{ cm.}$$

Since q is $+2\frac{1}{7}$ cm., the image is in front of the mirror and therefore real.

$$\frac{\text{Size of image } II'}{\text{Size of object } OO'} = \frac{q}{p} = \frac{15}{7} \times \frac{1}{5} = \frac{3}{7} \qquad \text{or} \qquad II' = \tfrac{3}{7} OO'$$

EXAMPLE 2. We shall now use the formulas to calculate the position and size of an image given by the CONVEX mirror on page 168.

$$p = OA = +3 \text{ cm.}; \qquad f = FA = -\tfrac{3}{2} \text{ cm.}; \qquad q = IA = ?$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \text{or} \qquad \frac{1}{q}=\frac{1}{f}-\frac{1}{p}=-\frac{2}{3}-\frac{1}{3}=-\frac{3}{3}=-1$$

$$\therefore \quad q = -1$$

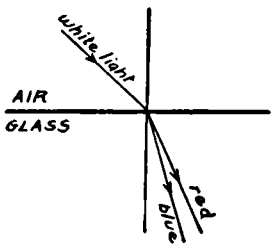
$$\frac{II'}{OO'}=\frac{q}{p}=\frac{1}{3} \qquad \therefore \quad \text{Size of image} = \tfrac{1}{3} \text{ size of object}$$

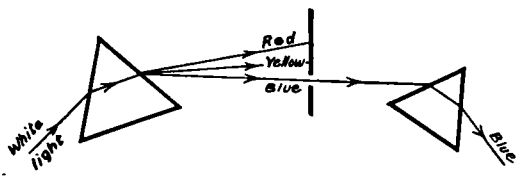
Since q is negative, the image is behind the mirror and, therefore, virtual.

REFRACTION OF LIGHT WAVES

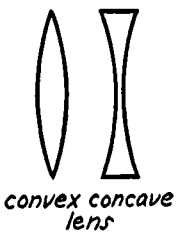
IT MAY seem peculiar that light waves which travel in a vacuum are slowed down when they travel in water, glass, etc. We showed, in the chapter dealing with waves, that when waves of length λ_1 and velocity v_1 move into a medium where their velocity is v_2, their wave-length is changed to λ_2 in accordance with $v_1/v_2 = \lambda_1/\lambda_2$. In light this ratio v_1/v_2 is called the *refractive index of the two media.* So we see that all light waves, whatever their wave-length, travel with the same velocity in a vacuum, but this is NOT the case in water or any other dense medium where the *shorter* waves are slowed down *more* than the longer rays. The velocity of light in air or any gas at ordinary pressures is so nearly the same as in a vacuum that we can neglect this small difference without making any appreciable error.

Now we distinguish the various light waves by the color of the light; in red light the waves are longer than in blue light. Blue rays are slowed down more in glass, water, etc., than are the red rays. Thus when a beam of white light enters a denser medium at an angle, the waves and hence the colors are separated or dispersed. All the rays of light are bent when they enter water, but the blue rays are bent the most. If we wish to observe this effect in air we must bend or refract the rays twice by using a prism of glass. It was by such an experiment as this that Newton first broke up white light from the sun into its separate colors, i.e., into its spectrum. He tried to break up these separate colors further by using a second prism, but without success. This separating of light into its different colors is called DISPERSION.

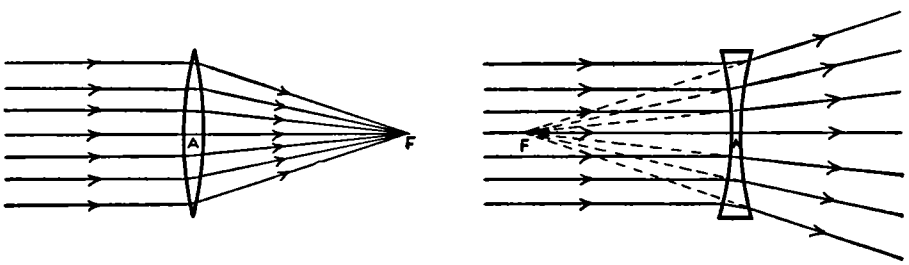




LENSES: Lenses may be considered as similar to two thin prisms of glass put end to end. The two types of lenses are known as *convex* and *concave*. A CONVEX lens is thicker at the middle than at the edges, while a CONCAVE lens is thinner at the middle than at the edges. The faces of the lenses are not plane; the exact shape depends on what the lens is to be used for, and often presents a very difficult optical problem. We shall assume that the surfaces of the lenses are spherical and that the lenses themselves are relatively thin.



Suppose a beam of parallel light rays falls on a lens as shown. If the rays are parallel to the axis AF, the convex lens will cause all these rays to converge to a point F on the axis called the focus of the lens. The parallel rays, falling on the concave lens, will be made to diverge. They will appear to come from a point F on the axis called the *focus*. For this reason a convex lens is often called a *converging* lens, while a concave lens is called a *diverging*

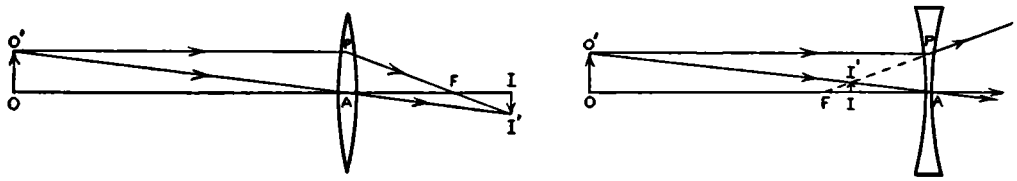


lens. The distance FA is called the focal length of the lens. Now, if a ray of light passes through the center of the lens A, it will come out in the same direction as it entered but slightly displaced.* This difference may be neglected when the lens is thin.

In order to find the position of the image of an object OO' placed on the axis OAF, we shall draw

a) a ray $O'P$ from the head of the object parallel to the axis (this ray, after passing through the lens, passes through the focus F, or, in the case of the concave lens, appears to come from the focus);

b) a second ray $O'A$ from the head of the object and passing through the center of the lens (this ray goes through undeflected).

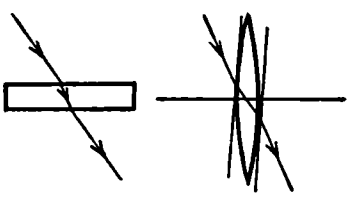


The intersection of these two rays gives the position of the image II'. Notice that the image formed by the convex lens is real (the rays actually pass through II' and the image can be focused on a sheet of paper), whereas the image formed by the concave lens is virtual. The image formed by the convex lens is real, inverted, and diminished, while the image formed by the concave lens is virtual, upright, and diminished.

There is a formula which gives the relation between object distance, image distance, and the focal length of the lens.†

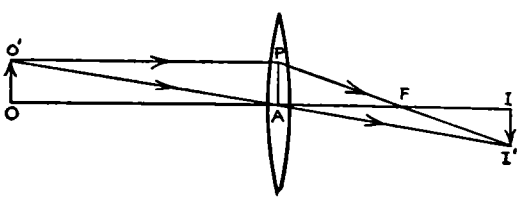
$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f},$$

* A ray of light passing through a parallel-sided plate of glass will emerge parallel to its original direction. Now, if the surfaces of the lens have the same curvature, the tangents to the surfaces at the points where the light passing through the center of the lens enters and emerges are parallel. Thus a ray passing through a thin lens will emerge parallel to the direction in which it enters.



† To prove the formula $1/p+1/q=1/f$, we shall take the case of a convex lens forming a real image. This is the easiest case. The triangles OAO', IAI' are similar.

$$\therefore \quad \frac{OO'}{II'}=\frac{OA}{IA}=\frac{p}{q}$$



The triangles AFP, IFI' are similar.

$$\frac{PA}{II'}=\frac{AF}{FI}=\frac{f}{q-f}$$

[Footnote continued on facing page.]

where p = distance of object from center of lens, q = distance of image from center of lens, and f = focal length of lens. To use this equation for both convex and concave lenses we must give a rule for the signs of the quantities p, q, and f. Each of the distances p, q, and f is positive in the case of a convex lens forming a real image. The focal length of a convex lens is positive, that of a concave lens is negative. The formula relating the size of the image to that of the object is in the case of lenses:

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{image distance}}{\text{object distance}} = \frac{q}{p}$$

Let us apply these formulas to the two types of lenses.

EXAMPLE 1. CONVEX LENS: Find the position and size of the image produced by a convex lens whose focal length is $\frac{4}{3}$ cm., when the object is placed 4 cm. from the lens.

$$p = OA = +4 \text{ cm.}; \qquad f = +\tfrac{4}{3} \text{ cm.}; \qquad q = ?$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \text{or} \qquad \frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{3}{4}-\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$$

$$\therefore \quad q = +2 \text{ cm.}$$

The image is real (shown by the + sign) and is 2 cm. from the lens on the opposite side of the lens from the object.

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{q}{p} = \frac{2}{4} = \frac{1}{2}$$

The image is half as large as the object.

EXAMPLE 2. CONCAVE LENS: Find the position and size of the image formed by a concave lens whose focal length is $\frac{4}{3}$ cm., if the object is placed 4 cm. from the lens.

$$p = OA = +4 \text{ cm.}; \qquad f = -\tfrac{4}{3}; \qquad q = ?$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{q}=\frac{1}{f}-\frac{1}{p}=-\frac{3}{4}-\frac{1}{4}=-\frac{4}{4}=-1$$

$$\therefore \quad q = -1 \text{ cm.}$$

Since the sign is negative the image is virtual. It is 1 cm. from the lens on the same side of the lens as the object.

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{q}{p} = \frac{1}{4}$$

The image is only a quarter as large as the object. The sign of q in the latter equation does not matter since we are only interested in the magnitude of the image.

But $OO' = PA$.

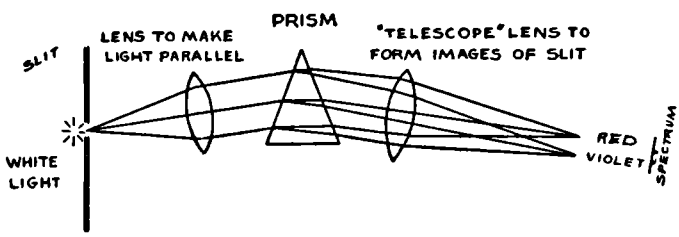
$$\therefore \quad \frac{OO'}{II'} = \frac{PA}{II'} \qquad \text{or} \qquad \frac{p}{q} = \frac{f}{q-f} \qquad \therefore \quad pq - pf = qf$$

Dividing all the terms of the equation by pqf:

$$\frac{pq}{pqf} - \frac{pf}{pqf} = \frac{qf}{pqf} \qquad \text{or} \qquad \frac{1}{f} - \frac{1}{q} = \frac{1}{p} \qquad \therefore \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

(The proof for the concave lens is similar, but the distance FA must be put equal to $-f$ and the distance IA to $-q$.)

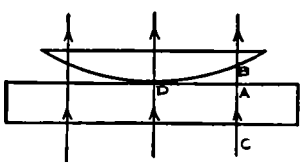
LENSES like prisms will disperse white light into its various colors. This means that the focal length of the convex lens for red light is different from the focal length for blue light. Which is greater in the case of a convex lens, the focal length for blue light or that for red light?§ The image tends to become colored by dispersion.* The effect can be minimized by the use of several lenses properly placed. If a clear image of the spectrum of white light is to be obtained with a prism, a bundle of parallel rays must be sent through the prism. The parallel bundle emerging is then collected by a lens. Distinct colored images of the source are obtained.



Dispersion of a parallel beam of white light by a prism into a spectrum. A prismatic spectroscope

INTERFERENCE IN LIGHT

IN THE seventeenth century Newton performed an experiment which showed the interference of light waves, though he did not interpret his experiment in this way. If a convex lens placed on a flat glass surface is held in front of a light, a series of small colored rings is seen around the point of contact. A ray of light coming in the direction CA will be partly transmitted at B and partly reflected. The reflected portion will be again partly reflected at A and will then join rays at B which have not suffered reflection. The reflected ray has to travel farther by a distance equal to twice AB than the ray going right through to B. Now if $2AB$ is any whole number of wave-lengths λ, a crest from the one wave going right through will fall on a crest from the other which is twice reflected. Hence B will be a bright spot, or rather a portion of a bright ring. If, however, $2AB$ is an odd number of half wave-lengths, $\lambda/2$, $3\lambda/2$, etc., a crest of one wave will meet a trough of the other. The result will be darkness at that point. If D is the point of contact of the lens and the plate, the separation of the lens and plate increases as we go out from D. Hence a series of bright and dark rings will be seen around D. Since the wave-length of red light is greater than the wave-length of blue light, the rings are colored. For a given ring, which color will be on the inside of the ring, red or blue?§§



A similar sort of interference pattern is produced when an oil film spreads

* An interesting discussion of this topic will be found in Pendray, *Men, Mirrors and Stars.*

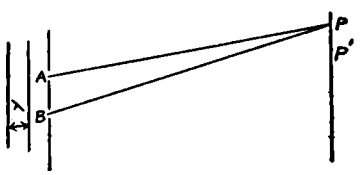
on water. Interference takes place between a wave reflected from the top and one reflected from the bottom of the oil film.

DIFFRACTION OF LIGHT

HAVE YOU ever wondered how the colored rings which encircle a bright light on a misty night are produced? Similar rings, called coronas, are frequently seen around the moon. They are produced by diffraction. Water droplets in the atmosphere act as the diffracting obstacles. From what we have already learned about diffraction, we know that, for a given opening or small obstacle, the greater the wave-length the greater the bending of the waves. Which color, red or blue, would you expect to be on the outside of the corona?§ The greater the angular radius of the circles, the smaller are the diffracting water droplets. Thus a decreasing corona circle implies that the droplets are getting larger. Hence rain is probable. The size of the diffracting water droplets is several times as large as the wave-length of light waves. When the diffracting obstacles are smaller than the wave-length of the light waves, the effect is quite different; the short waves, blue, are much more diffusely scattered than the long waves. Thus the sun's light, when it has to pass through a lot of the atmosphere as at sunrise or sunset, appears red; the blue light has been removed by scattering, giving rise to the blue color of the sky.

DIFFRACTION by an obstacle was first observed and reported by the Jesuit father, Francesco Gramaldi, about 1650. His report appears to have been overlooked because the wave nature of light was not generally recognized until after the work of Young and Fresnel about 1800. Since the wave-lengths composing visible light are so small, the amount of bending due to diffraction by an obstacle is small. Good instruments have to be used to detect it. It is for this reason that light is said to travel in straight lines.

IN YOUNG'S experiment, light was passed through two slits close together and the interference pattern produced at some distance was observed. We have already discussed this problem for waves in general. We found that if $(BP-AP)=\lambda$ or 2λ, etc., then at P a crest from one wave falls on a crest from the other wave and there is brightness at P. For another point P', if $(BP'-AP')=\lambda/2$ or $3\lambda/2$, etc., a crest from one wave will fall on a trough from the other and darkness will result at



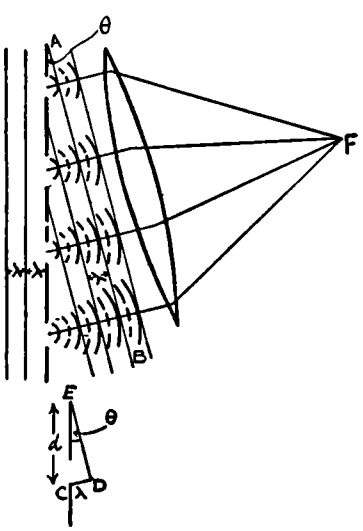
§ Answer to question on the previous page: The focal length for red light.

§§ Answer to question on the previous page: Blue.

this point P'. At the various points along the screen there will be bright and dark spots.

Fresnel's work was more extensive than that of Young, but it is interesting to note that at the time, 1818, when Fresnel wrote on diffraction he regarded light waves as longitudinal. This of course makes no difference in the explanation of the various phenomena. The discovery of the fact that light waves can be polarized finally settled what had been a big dispute on the nature of light waves. The polarizers, corresponding to the slits in the previous discussion on this subject, are called Nicol prisms and are made of calcite crystals.

IN 1821 Fraunhofer made his discovery of the diffraction grating. A diffraction grating consists of a number of fine slits equally spaced and very close together. Modern gratings are made by scratching fine lines on a surface of glass with a diamond point. Where the scratches occur the light does not pass through, but between the scratches the light passes through as through a fine slit. Suppose plane waves are incident on the grating. Then each of the openings or slits will send out waves. Let us consider a resultant wave front such as AB. The waves from each of the openings will arrive in phase along AB, if the consecutive distances of AB from the openings are each greater by one whole wave-length. This is shown by the waves coming from each of the openings. If the wave front AB is collected by a lens, a bright point will be obtained at the focus F of the lens. For a single opening the excess distance between it and the next opening is one wave-length: $CD=\lambda$, but $CD=EC \sin \widehat{CED}=d \sin \theta$, so that if $d \sin \theta=\lambda$, the wave front AB will produce brightness. For some angle θ_2 such that $d \sin \theta_2=2\lambda$, there will be brightness, etc.



If, however, we consider another wave differently oriented from AB such that if an opening produces a crest the next one produces a trough on the wave front, etc., the resultant amplitude will be zero and darkness will result: For some angle θ_1, such that $d \sin \theta_1=\lambda/2$ or $3\lambda/2$, etc., there will be darkness.

We see that the larger λ is, the larger is $\sin \theta$ or θ, i.e., the more the wave is

§ Answer to question on the previous page: Red.

diffracted. The red is on the outside and the blue on the inside of the diffraction pattern. The distance between the openings, d, must be very fine. Modern gratings can be ruled so that there are 30,000 lines to the inch. Then $d = 1/30{,}000$ in. $= .000033$ in. Diffraction gratings are usually used for measuring wave-lengths accurately. The conventional unit for wave-lengths is the Ångstrom unit.

$$1 \text{ Å} = 10^{-8} \text{ cm.}$$

Yellow light has a wave-length of 0.00005890 cm., i.e., $5{,}890 \times 10^{-8}$ cm. or 5,890 Å. The wave-length of red light ranges around 6,500 Å, while that of violet light is around 3,500 Å.

PROBLEMS

1. Assuming that light is a wave motion, on what property of a wave (amplitude, frequency, or wave-length) does the brightness of light depend?

2. A horizontal ray of light strikes a mirror set at 45° to the horizontal.
   a) Find what angle the reflected ray makes with the horizontal?
   The mirror is turned through 10° so that it now makes an angle of 35° with the horizontal.
   b) Through what angle is the reflected ray turned?
   c) Find what angle the reflected ray now makes with the horizontal.

3. You are standing 25 cm. away from a convex mirror whose focal length is 25 cm.
   a) Find the position of your image.
   b) Describe the image produced (real or virtual, upright or inverted, magnified or diminished).

4. *a*) Is it possible to produce a real image of an object using a convex mirror?
   b) Is it possible to produce a magnified image of an object using a convex mirror?

5. A Newtonian telescope uses a concave mirror to focus the light of a star. If the radius of curvature of the mirror is 3 m.,
   a) How far from the concave mirror will the image of the star be?
   b) Will this image be real or virtual?

6. Does a concave mirror bring all the various colors of sunlight to the same focus?

7. If an electric lamp 6 cm. in height is placed 20 cm. in front of a concave mirror whose focal length is 30 cm.,
   a) Find the position and size of the image.
   b) Describe the image (real or virtual, upright or inverted, magnified or diminished).

8. The lamp of problem 7 is placed 30 cm. from the mirror. Describe the image produced.

9. The lamp of problem 7 is placed 60 cm. in front of the mirror.
   a) Find the position and size of the image.
   b) Describe the image.

10. The lamp of problem 7 is placed 90 cm. in front of the mirror.
   a) Find the position and size of the image.
   b) Describe the image.

11. Draw diagrams for determining the position, etc., of the images produced in the preceding 4 problems. (Draw on a scale of 1 in 10.)

12. If you want to see a magnified image of your face in a mirror,
   a) What sort of mirror, plane, concave, or convex, would you use?
   b) Would you want a real or virtual image?

13. Does a convex lens which is thick at the center have a greater, less, or the same focal length as one which is thin at the center?

14. A candle whose flame is 2 cm. high is placed 9 cm. from a concave lens which has a focal length of 3 cm.
   a) Find the position and size of the image.
   b) Describe the image.
   c) Draw a diagram showing the position of the image.

15. *a*) Is it possible for a concave lens to produce a real image of an object?
   b) Is it possible for a concave lens to produce a magnified image of a real object?

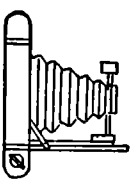
16. A convex lens whose focal length is 20 cm. forms an image of the sun.
   a) How far from the lens is the image?
   b) Are the various colors of sunlight brought to the same focus?

17. You may feel the need of a lens for magnifying the print in this book:
   a) Would you use a convex or a concave lens?
   b) Would you want a real or a virtual image? Why?

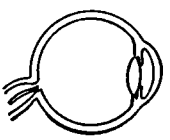
18. Suppose you hold a convex lens whose focal length is 10 cm. a distance of 6 cm. from the book.
    a) Where is the image of the print and how much is it magnified?
    b) Describe the image.
    c) When the lens is 10 cm. from the book, where is the image?
    d) Could you read the print with the lens in this position?

    The lens is moved to a distance of 20 cm. from the book.
    e) Where is the image of the print formed?
    f) Describe the image formed.
    g) As the lens is moved farther away from the book, does the image of the print move farther away or closer to the lens?

19. A photographic camera consists of a light-tight box with a movable lens in front which forms a real image on a photographic film at the back of the camera.



    a) Is the lens convex or concave?
    b) Where should the photographic film be with respect to the lens when photographing far-distant objects?
    c) When photographing nearby objects must you move the lens toward or away from the film?



20. The human eye resembles a photographic camera. The convex eye lens forms a real image of objects on a sensitive retina at the back of the eye. In order to focus either near or distant objects on the retina, there are muscles which can change the curvature of the lens of the eye and thus change its focal length.
    a) When viewing near objects, must the lens have a greater or shorter focal length than when viewing distant objects? When this "accommodation" of the eye becomes faulty, glasses must be worn.

    A short-sighted person is one who can see near objects distinctly but not distant ones.
    b) Are distant objects focused in front or behind the retina in the case of a short-sighted person?
    c) Should this person wear glasses with convex or concave lenses?

    A long-sighted person is one who can see distant objects distinctly but not near ones.
    d) For this person are near objects focused in front of or behind the retina?
    e) Should this person wear glasses with convex or concave lenses?

21. You photograph a friend of yours while he is lying on the ground with his feet toward you.
    a) Why do his feet appear unduly large in the picture?
    b) Is it for this reason that, if you tilt the camera up when photographing a tall building, it will appear to be falling over backward in the picture?

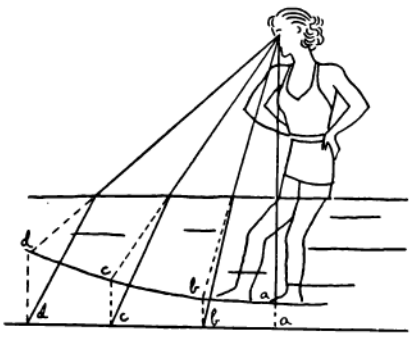
22. A simple telescope may be made of two lenses: a convex objective and a convex eyepiece. The objective forms a real image of a distant object which is then magnified by the eyepiece. Suppose the objective of a telescope has a focal length of 50 cm. and the eyepiece a focal length of 2 cm. The distance between the objective and the eyepiece is $51\frac{1}{15}$ cm. You look through this telescope at a building 20 m. high and 500 m. away.
    a) Where does the objective of the telescope form its image?
    b) Find the size of this image.
    c) Find the position and size of the final image, the one which you see when you are looking through the eyepiece.

23. When you hold your handkerchief close to your eye and look through it at a bright light, to what is the effect that you see due (reflection, refraction, or diffraction)?

24. *a*) Will the light received from a star approaching the earth have a shorter or longer wave-length than if the star were at rest?
    b) Will the light be slightly more blue or slightly more red when the star is approaching the earth?
    c) If the star is moving away from the earth, will the light received from it be slightly more blue or more red than if the star were at rest?

25. It has been found that the equatorial zone of the sun rotates eastward once in approximately twenty-five days. Would the light from the eastern edge of the sun be slightly shifted to the blue or red? (The western edge is receding.)

26. The wave-length of a red line received from hydrogen gas is 6,600 Å, while the wave-length of a blue line is 4,000 Å.
    a) Find the frequency of these two colors.
    b) Which is refracted more on entering a block of glass—the red or the blue light?
    c) Are the wave-lengths or the frequencies of the light changed when it enters the glass?



27. Does refraction cause a pond to appear more or less deep than it really is?

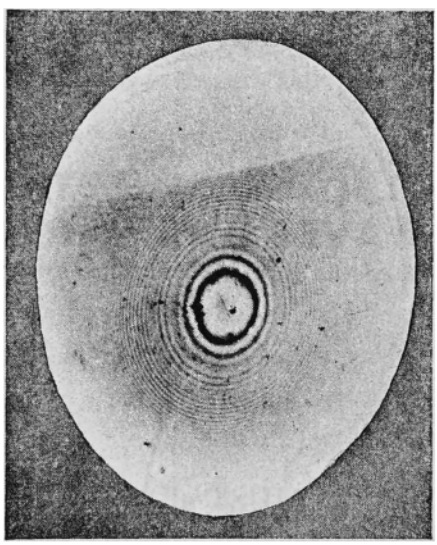
28. White light is broken up into a spectrum by two diffraction gratings. The one, A, has a large number of lines per cm., while the other, B, has a small number of lines per cm.

a) Which grating, A or B, diffracts the whole spectrum by the greater amount?

b) Which separates the red and blue parts of the spectrum the most, i.e., produces the greatest dispersion?

c) Which color, red or blue, is diffracted the most?

29. A convex lens with a radius of curvature of 100 cm. is placed on a flat glass plate. When a light is held below the plass plate, Newton's Rings are seen round the point of contact.



a) Find the distance between the lens and the glass plate at the first blue ring (wave-length of blue light is 4,000 Å.).

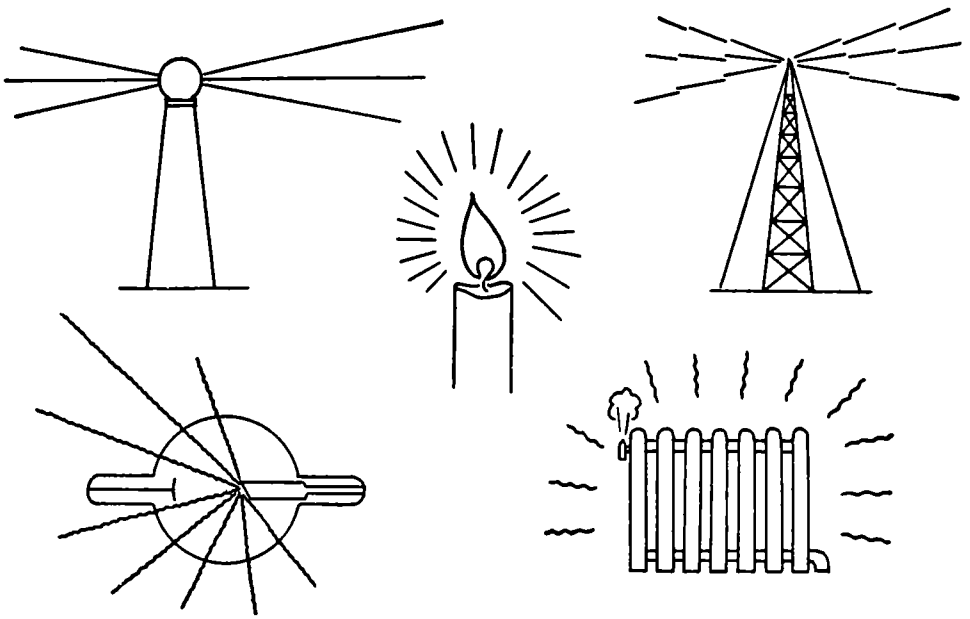
b) Find the distance between the lens and the plate at the first red ring (wave-length of red light is, say, 6,400 Å.).

c) Using the equation $h=d^2/2R$ given on page 55, find the radius of the first blue ring; of the first red ring. (This radius corresponds to d, the separation of the lens and the plate to h, and the radius of the lens to R.)

30. Use the general equation you developed in the last chapter for the Doppler principle to solve this problem. The wave-length of a spectrum line coming from a star is changed from 6,000 to 6,001 Å due to the motion of the star.

a) Is the star moving toward or away from the earth?

b) Find the velocity of the star with respect to the earth. (The velocity of light is 3×10^{10} cm. per sec.)



ELECTROMAGNETIC RADIATIONS: WAVES AND PARTICLES

All black body radiations,
All the spectrum variations,
All atomic oscillations
Vary as *hn*.

YOU MUST all have at one time or another sat in front of a hot stove and watched it turn dull red and then cherry red. Did you wonder if some invisible radiation was coming from the stove before it became red hot? The eye can only see electromagnetic radiation whose wave-length is between 7,700 and 3,800 Å. Below 3,800 Å are the ultra-violet radiations and above 7,700 Å are the infra-red radiations. It is these infra-red (long wave-length) radiations which the hot stove is sending out before it becomes red. Actually the radiation coming from the hot stove depends not only on the temperature of the stove but also on the nature of its surface. You will recall that a thermos bottle was silvered on the inside to cut down radiation losses because a bright object emits less radiation than does a black one at the same temperature.

CONTINUOUS SPECTRA

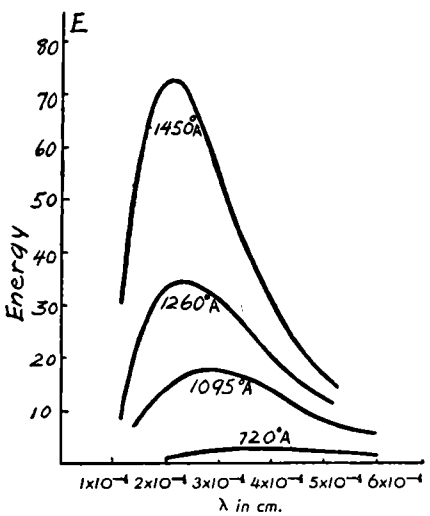
LET US measure the radiation which comes from a furnace at various temperatures. We shall be dealing with radiation coming from a perfectly black object, so-called *black-body radiation.* Such radiation depends only on the temperature of the object, and forms a *continuous spectrum.* We find that the TOTAL RADIATION COMING FROM THE FURNACE VARIES AS THE FOURTH POWER OF THE ABSOLUTE TEMPERATURE. This result is known as STEFAN'S LAW:

$$E \propto T^4 \qquad \text{or} \qquad E = \text{a constant} \times T^4, \text{ i.e., } E = K \times T^4$$

If E_1 is the energy coming from the furnace when its temperature is $T_1°$ A. and E_2 the energy at temperature $T_2°$ A., then $E_1 = KT_1^4$, $E_2 = KT_2^4$, or $\frac{E_1}{E_2} = \frac{T_1^4}{T_2^4}$.

EXAMPLE. Tyndall, about 1880, found that 11.7 times as much energy came from a platinum wire at 1,200° C. as from the same wire at 525° C. According to Stefan's Law, this ratio of energy should be $\frac{(1{,}200+273)^4}{(525+273)^4}$, which is 11.6. The agreement of experiment with Stefan's Law is very good considering the measuring instruments available at that time.

LET US now break up this radiation into its separate wave-lengths by means of a prism using the apparatus shown on page 174. Our lenses and prism will have to be made of fluorspar which is transparent to infra-red radiation. We shall measure the intensity of the radiation in these wave-lengths and record our results on a graph. If this is repeated for several different furnace temperatures, we see from the graphs that the hotter the furnace the shorter the wave-length of the radiation which has maximum intensity. (Maximum intensity, of course, means the ordinate of the peak of the curve.) There is a simple relationship, first given by Wien, relating the wave-length having the maximum intensity λ_m and the absolute temperature T of the furnace which is giving out the black-body radiation.



Distribution of energy in radiation from a "black body"

WIEN'S DISPLACEMENT LAW: $\lambda_m T = a$ constant. Taking the data from the graphs we find that $\lambda_m T = 0.29$.

Temperature in ° Absolute	λ_m in cm.	$\lambda_m T$
1,450	0.00020	0.29
1,260	.00023	.29
1,095	.00027	.29
720	0.00041	0.29

We are dealing with a CONTINUOUS SPECTRUM which is characteristic of a radiating solid or gas at high pressure; for this continuous spectrum we have the two important laws just discussed.

IF WE examine the radiation, or light, which comes from a radiating gas which is at low pressure, as is found in a neon sign, we find a series of distinct colors, i.e., bright lines,* which are separated from one another. This type of spectrum is called a BRIGHT-LINE SPECTRUM. The color or wave-length of the bright lines depends on the type of gas used. We also call this bright-line spectrum an ATOMIC SPECTRUM since it is characteristic of radiating atoms. Molecules, on the other hand, give what is known as a BAND SPECTRUM; actually the bands of color are a series of bright lines very close together. Molecular spectra are very much more complicated to analyze than atomic spectra.

IF WE allow the light which gives a continuous spectrum to pass through a relatively cool gas or vapor at low pressure before it reaches the spectroscope, the spectrum produced will be continuous except for dark lines where there would be bright lines if the gas or vapor at low pressure were the only radiating source. Such a spectrum is known as a DARK-LINE ABSORPTION SPECTRUM.

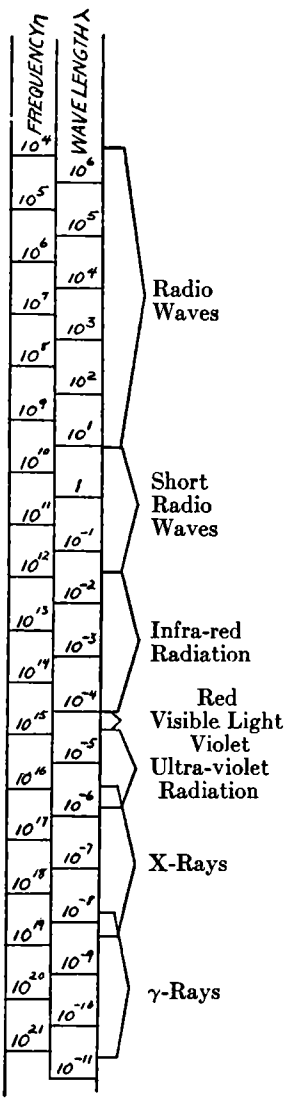
LIGHT QUANTA OR PHOTONS

LET US now return to our continuous spectra and the graphs showing the intensity of the radiation in different wave-lengths. A great deal of effort was put into trying to obtain, from purely theoretical considerations, an equation which fitted these graphs. About 1900 Planck finally succeeded, but in doing so he made some radical departures from the accepted physical principles of the time. He assumed that atoms radiate energy discontinuously in bundles of amount hn, where n is the frequency of the radiation and

* Our spectroscope produces colored images of the slit behind which the radiating source is placed. Usually the slit is made narrow so that the separate colors appear as bright lines (see p. 174).

h is a universal constant known as PLANCK'S CONSTANT: $h = 6.55 \times 10^{-27}$ erg sec.

It was Planck's idea that the source of discontinuity was in the radiating atoms, not in the radiation itself. In 1905 Einstein put forward the idea that the discontinuity was in the radiation itself, i.e., that the energy in light (or any electromagnetic radiation) was in bundles of amount hn. These bundles of energy are known as LIGHT QUANTA OR PHOTONS. Thus electromagnetic radiation is assumed to be atomic in nature just as is electricity or matter. Einstein made still another radical assumption, i.e., a photon could transfer all of its energy, hn, to an electron. These assumptions were made in order to interpret some experimental results obtained on the photoelectric effect which the wave theory of light was unable to interpret.



Electromagnetic spectrum radiations traveling with velocity of light 3×10^{10} cm. per sec.

You may recall the photoelectric effect from the experiment in which ultra-violet light was allowed to fall on a clean zinc plate. Photoelectrons were given off. If the ultraviolet lamp is brought closer to the zinc plate, the intensity of the light falling on the plate is increased. You would expect an increase in both the number and the velocity (or energy) of the photoelectrons. This is what the wave theory of light would predict. Actually what happens is that the number of photoelectrons given off is increased, but there is no increase in their velocity or energy. To interpret this required the genius of an Einstein.

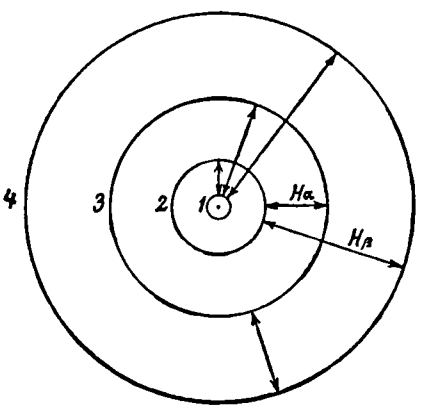
According to Einstein, a photon with energy hn falling on the zinc plate gives up its energy to an electron. The electron then proceeds to use part of this energy, in amount E_a, to escape from the zinc plate. The remainder is kept by the electron as kinetic energy, $\frac{1}{2}mv^2$. Thus the photoelectron comes out of the zinc plate with a kinetic energy, $\frac{1}{2}mv^2$, equal to $hn - E_a$. The only way in which we may increase the velocity and energy of the photoelectrons coming from the zinc plate is to increase n, the frequency of the light falling on the plate. This means using ultra-violet light of shorter wave-length.

The closer the source of ultra-violet light is brought to the zinc plate, the greater the number of photons which fall on the plate, and thus the greater the number of photoelectrons which are given off.

SPECTRUM OF THE HYDROGEN ATOM

ALL OF this work had been done before Bohr brought forward his picture of the atom in 1913. We shall now consider the Bohr picture of the hydrogen atom and see how he accounted for the bright-line spectrum of hydrogen.

The hydrogen atom has a nucleus built up of a single positive charge, a proton, and one planetary electron. The planetary electron is supposed to revolve around the nucleus in such a way that the centrifugal force due to the revolution balances the pull of the proton on the electron. The electron cannot revolve in any circle but must revolve in certain prescribed circles called *orbits, stationary states,* or *energy levels.** There is a definite amount of energy associated with the electron in each of these orbits. It must be emphasized that the atom *does not radiate* any energy when the electron is revolving in one of these orbits, but *only when the electron falls from a higher to a lower level, i.e., falls toward the nucleus.*† Suppose E_2 represents the energy of the electron in orbit 2 and E_1 the energy in orbit 1. When the electron falls from orbit 2 to orbit 1, it gives off light of a single frequency n where $E_2 - E_1 = hn$. We cannot possibly give any of the mathematics necessary for the calculation of the energy of the electron in the various orbits of the hydrogen atom, but shall state that an electron falling from orbit 2 to orbit 1 gives out light of a frequency such that $n = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$.‡ The value of R for hydrogen is 3.29×10^{15}.



The original Bohr model of the hydrogen atom

* Later the theory was modified so that electrons were regarded as revolving in ellipses round the nucleus as a focus (cf. sun and the planets).

† This is contrary to the classical theory of electrodynamics developed by Maxwell and others.

‡ $E_2 = -\frac{Rh}{2^2}$ and $E_1 = -\frac{Rh}{1^2}$, so that $E_2 - E_1 = Rh\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$.

EXAMPLE. Calculate the frequency and wave-length of the light emitted when the electron in hydrogen falls from orbit 2 to orbit 1.

$$n = 3.29\times10^{15}\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = 3.29\times10^{15}(1-\tfrac{1}{4}) = 3.29\times10^{15}\times\tfrac{3}{4} = 2.47\times10^{15} \text{ vibrations per sec.}$$

From the equation $v = n\lambda$ we may find λ. Here v is the velocity of light, 3×10^{10} cm. per sec., and is usually denoted by c.

$$\therefore\quad \lambda = \frac{c}{n} = \frac{3\times10^{10}}{2.47\times10^{15}} = 1.215\times10^{-5} = 1,215\times10^{-8} \text{ cm.}$$

$$= 1,215 \text{ Å } (1 \text{ Å} = 10^{-8} \text{ cm.})$$

This radiation is in the far ultra-violet and is known as the first line of the Lyman series for hydrogen. Is this radiation visible?§

The second line of this series would be emitted when the electron drops from orbit 3 to orbit 1.

Frequency of this radiation:

$$n = 3.29\times10^{15}\left(\frac{1}{1^2}-\frac{1}{3^2}\right) = 3.29\times10^{15}\times\tfrac{8}{9} = 2.95\times10^{15} \text{ vibrations per sec.}$$

Wave-length of this radiation:

$$\lambda = \frac{c}{n} = \frac{3\times10^{10}}{2.95\times10^{15}} = 1.015\times10^{-5} \text{ cm.} = 1,015 \text{ Å}$$

The electron in hydrogen is normally in the lowest orbit, orbit 1, and is raised to a higher orbit by absorbing energy. A means of doing this is to put hydrogen gas in an electrical discharge tube.* When we pass an electrical discharge through the gas, the light we see coming from the hydrogen is that sent out when the electron falls from outer orbits into orbit 2. This series of wave-lengths, first calculated by Balmer, is known as the *Balmer series.*

Problem: Calculate the frequency and wave-length of the first three lines of the Balmer series.

The first line H_α is due to an electron falling from orbit 3 to orbit 2, the second line H_β is due to an electron falling from orbit 4 to orbit 2, and the third line H_γ to an electron falling from orbit 5 to orbit 2.

Answer:

	Frequency n	Wave-Length λ	Color
H_α	4.56×10^{14}	6,560 Å	Red
H_β	6.17×10^{14}	4,860	Blue
H_γ	6.91×10^{14}	4,340	Violet

Of course any one atom of hydrogen is radiating only one wave-length at any instant, but some other atom will be giving out a different wave-length (at the same time), so that, in a discharge tube with its billions of atoms,

* This is done with neon in a "neon-sign" tube.

every wave-length of the hydrogen atom will be radiated. The general equation for the frequency of radiation given out by the hydrogen atom is

$$n = 3.29 \times 10^{15}\left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right)$$

For the Balmer series $k_1 = 2$ and k_2 has the values 3, 4, 5, etc., according to the orbit from which the electron falls.

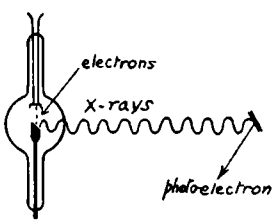
Problem: Suppose the electron falls into orbit 1 from such a far-distant orbit that k_2 is so large that $1/k_2^2$ is negligible. What is the frequency and the energy of the light quanta given out? (*Answer:* 3.29×10^{15} vibrations per sec.; 21.6×10^{-12} ergs.) This energy of 21.6×10^{-12} ergs would be sufficient to remove the electron of the hydrogen atom from orbit 1 to a far-distant orbit or completely outside the atom, i.e., to ionize the atom.

X-RAYS

WHEN WE proceed from hydrogen to the heavier atoms, the inner orbits, or shells as we previously called them, are filled with electrons. It is the outermost electron or electrons which give rise to the optical or visible spectrum. If we are by some means able to remove an electron from one of the orbits near the nucleus in a heavy atom, an outer electron will fall into this orbit and a photon having a large energy will be given out. The frequency of the radiation emitted is quite large. This type of radiation is known as *X-ray* radiation. The X-rays emitted by any one element have definite wavelengths and frequencies different from those of other elements, and are therefore called CHARACTERISTIC X-RAYS. In order to produce characteristic X-ray radiation, we must remove an inner electron from an atom. This may be done by other X-rays or by electrons having a high velocity. We stated earlier that X-rays are produced when high-speed electrons are suddenly stopped. In this case not only are *characteristic X-rays* given off but also a *continuous X-ray radiation.*

X-rays are capable of passing through the human body. They are, however, more absorbed by the heavy bone than by the comparatively less dense flesh. It is this difference in the absorption of X-rays by the light and heavy portions of the body that makes X-rays so important in medical work.

LET US consider the following process:



a) Suppose we put a *PD* of 30,000 volts across an X-ray tube. The electrons striking the target will have an energy of eV equal to $1.60 \times 10^{-19} \times 30{,}000$ joules, or 4.80×10^{-15} joules.

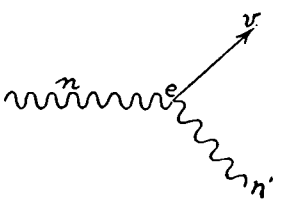
b) These electrons strike the target of the X-ray

§ Answer to question on the previous page: No.

tube where they produce photons of X-radiation some of which have this same energy, 4.80×10^{-15} joules. (Calculate the frequency of these photons, remembering that the energy hn of a photon is in ergs, not joules. *Answer:* 0.73×10^{19} vibrations per sec.)

c) These photons, falling on a zinc* plate, eject some photoelectrons some of which have this energy of 4.80×10^{-15} joules. Thus in (*c*) we obtain some photoelectrons from the zinc plate which have exactly the same energy as the filament electrons in (*a*) had when they struck the target of the X-ray tube. This energy has been carried undiminished by the X-ray photons. We think of a particle as able to carry energy, $\frac{1}{2}mv^2$, from one point to another without losing any on the way; but waves spread out in all directions and consequently their energy over any given area is less the farther we go from the source of the wave. Consider a stone dropped into a pond and the increasing size of the circles produced as we go from the original splash. We are thus compelled to consider the X-ray as having particle-like characteristics.†

Let us describe an experiment by Professor A. H. Compton which also supports this idea. He allowed X-rays to fall on a piece of carbon (any element of low atomic number would have done) and measured the energy of the secondary X-rays coming off the carbon block. These secondary X-rays were of lower frequency n' and hence lower energy than the primary X-rays. The difference in the energy of the primary and secondary photons of X-rays must have been given to a free electron in the carbon block. There had been a collision between a photon and an electron. The photon went off with less energy and momentum while the electron gained this energy and momentum. The collision is similar to a collision of two perfectly elastic billiard balls in which both energy and momentum are conserved.



Collision of a photon with a free electron

IF A photon has these particle-like characteristics, then surely we should be able to find the mass of a photon. To do this we must again appeal to Einstein, who obtained the relation that a particle whose mass is m gm. has intrinsic energy due to this mass of mc^2 ergs ($c=3\times10^{10}$ cm. per sec.). Thus 1 gm. mass is equivalent to 9×10^{20} ergs. Presumably, if we could somehow an-

* Almost any element would do.

† What would you say if a lifebuoy were dropped from a height of 6 ft. into the harbor at Boston. The resulting wave traveled to New York harbor. There a similar lifebuoy was lifted 6 ft. out of the water by the wave. We can guess your reaction, but is not this exactly what our X-rays did?

nihilate a 1 gm. mass, we should produce 9×10^{20} ergs of energy in the form of radiation. Since the product hn is energy in ergs and mc^2 is energy in ergs, we may get the mass of our photon by putting the two quantities equal to each other: $mc^2 = hn$.

$$\text{Mass of photon } m = \frac{hn}{c^2}; \qquad \text{momentum } (mc) \text{ of photon: } \quad mc = \frac{hn}{c}$$

But $c = n\lambda$ or $n = \frac{c}{\lambda}$, where λ is the wave-length of the radiation of frequency n. Substituting for n, we have

$$\text{Momentum of photon} = \frac{h}{c} \times \frac{c}{\lambda} = \frac{h}{\lambda}$$

This result applies to any electromagnetic radiation.

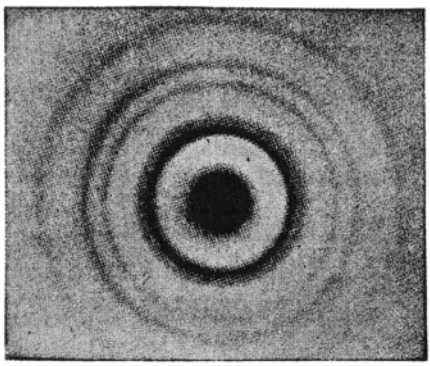
AFTER HAVING spent so much time on setting forth the particle characteristics of X-rays, you might naturally wonder if X-rays exhibit wavelike characteristics. Can they be diffracted? That is the crucial test for a wave. If we wish to use a diffraction grating, the openings of the grating must be very close together—much closer than can ever be humanly made. The distance between the openings, previously called d, must be of the order of 1 Å, or 10^{-8} cm. The idea occurred to Laue that natural gratings formed by the regular spacing of atoms in crystals might do. If we allow a narrow beam of X-rays to fall on a large number of powdered crystals in which the crystals are arranged in every possible direction, the X-rays will be diffracted according to the same equation that we derived for light waves (p. 176), i.e., for brightness or reinforcement of the waves $\lambda = d \sin \theta$, $2\lambda = d \sin \theta_2$, etc. The resulting diffraction pattern will be a series of circles, just as in the case of the rings around the moon.

WE HAVE shown that electromagnetic waves exhibit particle characteristics. Can we turn the table and show that what we call particles have wavelike characteristics? The first move in this direction was made by De Broglie in 1925. He showed that electrons of mass m and velocity v may be considered as waves having a wave-length $\lambda = h/mv$. (Notice the parallelism of this and the photon case, where we stated that the momentum of the photon is h/λ).

We should then expect electrons falling on powdered crystals to give a diffraction pattern similar to the pattern for X-rays. Experiments on diffraction of electrons were performed in 1927 by Davisson and Germer, of the Bell Telephone Laboratories, and also by G. P. Thomson, of Aberdeen. Thomson's experiments were similar to those we described for X-rays. The

diffraction pattern is shown alongside the one for X-rays. From the radius of the circles and the distance of the recording photographic plate from the powdered crystals it is possible to calculate λ. A value for λ was obtained

A typical X-ray (or ether wave) diffraction pattern produced by a powder or thin film in which the elementary minute crystals have been forced by the grinding and sifting, or else by the beating or rolling, to take a random distribution.



The corresponding diffraction pattern obtained when a beam of electrons (particles) passes through a thin gold foil in which the rolling or beating has given the elementary crystals again a random distribution.

which agreed with the relation $\lambda = h/mv$. Not only do electrons show these wavelike characteristics but atoms do also.

EXAMPLE. Let us calculate the wave-length of these electron waves for electrons which have fallen through a *PD* of 100 volts. The kinetic energy of the electrons will be the energy acquired by falling through 100 volts:

$$eV = 1.60\times10^{-19}\times100 \text{ joules} = 1.60\times10^{-10} \text{ ergs}$$

(This *KE* must be expressed in ergs since $\frac{1}{2}mv^2$ is in ergs when m is in gm. and v in cm. per sec.). To get mv for the electron we must use for the mass of the electron $m = 9\times10^{-28}$ gm. and do a little algebraic juggling.

$$\tfrac{1}{2}mv^2 = 1.60\times10^{-10} \qquad \therefore \quad v^2 = \frac{2\times1.60\times10^{-10}}{9\times10^{-28}} = 3.53\times10^{17}$$

$$\therefore \quad \text{Velocity of electron, } v = \sqrt{3.53\times10^{17}} = 5.95\times10^{8} \text{ cm. per sec.}$$

So that $$mv = 9\times10^{-28}\times5.95\times10^{8} = 53.55\times10^{-20} \text{ gm.}\times\text{cm. per sec.}$$

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{6.55\times10^{-27}}{53.55\times10^{-20}} = 1.22\times10^{-8} \text{ cm.} = 1.22 \text{ Å}$$

THUS WE may say that waves show particle-like characteristics and particles show wavelike characteristics. The wave theory of particles, founded by De Broglie, has been developed by Schrödinger, Dirac, and others into what is known as *wave mechanics.*

With this all too brief a review of what physics has done from Galileo to the present day, we must leave you. Let us hope you may yourself have felt some of the enthusiasm of those pushing the borderland of physics farther afield.

PROBLEMS

1. The wave-length having the maximum intensity in the spectrum of the sun is 4,700 Å. Find the temperature of the sun in °A. and °C.
2. An electric light with a tungsten filament attains a temperature of about 2,700° A.
   a) Assuming that the filament radiates as a black body, find the wave-length having maximum intensity in the spectrum from the electric light. The carbon filament in the old electric-light bulbs attained a temperature of about 1,800° A.
   b) Find the wave-length having the maximum intensity in this case.
3. Find the ratio of the total amount of energy coming from a tungsten filament electric lamp to that from a carbon filament lamp.
4. What is the ratio of the total amount of energy given off from a black body at a temperature of $2T°$ A. to that at $T°$ A.
5. The spectrum of the sun is a dark-line absorption spectrum, the dark lines being called the *Fraunhofer lines.*
   a) Does this suggest that the sun is surrounded by a layer of relatively cool gases?
   When the main disk of the sun is covered in an eclipse,
   b) What type of spectrum might you expect to see from the relatively cool gases?
6. *a*) Find the energy of an X-ray photon having a wave-length of 1 Å.
   b) Find the energy of a light photon which has a wave-length of 5,000 Å.
   c) Find the energy of a γ-ray photon which has a wave-length of 0.01 Å.
7. Can you explain why a greenhouse becomes warmer than the outside air?
   a) From where is the radiation which enters the greenhouse coming?
   b) When this radiation strikes the soil in the greenhouse, what happens to it?
   c) Is the radiation given off by the soil of the greenhouse greater or less in wave-length than the incoming radiation?
   d) Would you expect this radiation from the soil to be able to get through the panes of glass to the outside? (Glass is relatively opaque to very long and very short wave-lengths.)

8. Do radio waves travel with the same velocity as visible light?

9. Does a photon of a wireless wave have more or less energy than a photon of visible light?

10. Would you expect X-rays to be able to knock a photoelectron from a zinc plate?

11. Would you expect a photon of red light to be able to knock a photoelectron from a zinc plate?

12. *a*) Would the wave-length of proton waves be less or greater than the wave-length of electron waves if the proton and electron have the same velocity?
    b) What is the actual ratio of their wave-lengths?

13. Find the mass of a photon having a frequency of 10^{20} vibrations per sec.

14. Find (*a*) the velocity electrons acquire in falling through a *PD* of 300 volts, (*b*) the wave-length of the electron waves from electrons falling through a *PD* of 300 volts.

15. Assuming the wave nature of particles, find the wave-length of a 1 gm. mass moving with a velocity of 1,000 cm. per sec.

16. Suppose a twenty-dollar bill weighs 0.5 gm. If this amount of mass could somehow be annihilated,
    a) How much energy in ergs and joules would be produced?
    b) How much is this energy worth, assuming that 1 kw.-hr. of energy is worth 10 cents?
    c) On this basis, how much do you think this book is worth?

APPENDIX

BRITISH AND METRIC FUNDAMENTAL UNITS WITH ABBREVIATIONS

	British	Metric
Time	Second (sec.)	Second (sec.)
Length	Foot (ft.)	Centimeter (cm.)
Mass	Pound (lb.)	Gram (gm.)
Force	Poundal	Dyne

FORCE MAY BE EXPRESSED IN LB. WT. OR GM. WT. BUT MUST NOT BE USED IN NEWTON'S LAWS WITH THESE MASS UNITS

Force	lb. wt. (=32 poundals)	1 gm. wt. (=980 dynes)
Work $F\times s$ | KE $\frac{1}{2}mv^2$ | PE mgh	ft. poundal	dyne cm. or erg 1 joule $=10^7$ ergs
Work (use with care)	ft.×lb. wt.	gm. wt.×cm.
Power or rate of doing work	ft. lb. per sec. 1 horsepower (h-p.) = 550 ft. lb. of work per sec.	ergs per sec. 1 joule per sec. = 1 watt
	1 h-p. = 746 watts	

SIMPLE EQUATIONS WHICH MAY BE ENCOUNTERED IN THE TEXT

$v=\frac{s}{t}$. Then $s=vt$ or $t=\frac{s}{v}$

$s=\frac{1}{2}at^2$; $2s=at^2$; $a=\frac{2s}{t^2}$; $t^2=\frac{2s}{a}$ $\therefore$ $t=\sqrt{\frac{2s}{a}}$

Solve: $5x+3=2x+7$: $5x-2x=-3+7$

$3x=4$ or $x=\frac{4}{3}$

Solve the simultaneous equations:

$$x+2y=6$$
$$x-y=3$$

From the second equation: $x=y+3$.

Substitute this value for x in the first equation.

$$y+3+2y=6$$
$$3y=3, \quad \text{or} \quad y=1$$
$$x=y+3 \ =1+3=4$$

Solution is: $x=4$, $y=1$.

Eliminate t between $s=\frac{1}{2}at^2$ and $v=at$.

$$t=\frac{v}{a}, \qquad s=\tfrac{1}{2}a\frac{v^2}{a^2}=\tfrac{1}{2}\frac{v^2}{a}, \qquad \text{or} \qquad 2as=v^2$$

Symbols used for multiplication and division:

x multiplied by y may be written $x\times y$; $x\cdot y$; xy; yx; $y\cdot x$

x divided by y may be written $x\div y$; $\frac{x}{y}$; x/y; $x\left(\frac{1}{y}\right)$; xy^{-1}; $x:y$

Exponents:

$a^m\times a^n=a^{m+n}$: $a^6\times a^3=a^9$; $a^3\times a=a^4$

$a^m\div a^n=\frac{a^m}{a^n}=a^{m-n}$; $a^7\div a^2=a^5$

$a^6\div a^6=a^0$; but $a^6\div a^6=\frac{a^6}{a^6}=1 \quad \therefore \quad a^0=1$

$a^3\div a^6=\frac{a^3}{a^6}=\frac{1}{a^3}=a^{3-6}=a^{-3} \quad \therefore \quad \frac{1}{a^3}=a^{-3}$

$(a^m)^n=a^{mn}$

OUR NUMBER SYSTEM IS A PHYSIOLOGICAL ACCIDENT

(Ten fingers and ten toes)

It is often very convenient to express very large and very small numbers as powers of 10.

10^6 means 10 multiplied by itself 5 times (i.e., 10 used as a factor 6 times), or 1 with 6 zeros after it; i.e., a million, 1,000,000.

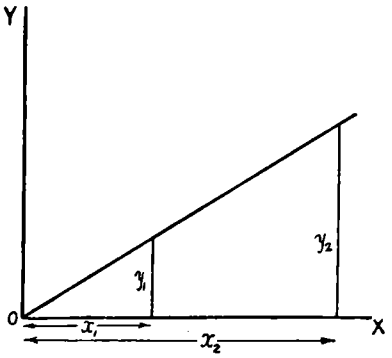
$10^4=10{,}000$	$10^{-4}=\frac{1}{10^4}=0.0001$
$10^3=\ 1{,}000$	$10^{-3}=\frac{1}{10^3}=0.001$
$10^2=\ \ 100$	$10^{-2}=\frac{1}{10^2}=0.01$
$10^1=\ \ 10$	$10^{-1}=\frac{1}{10^1}=0.1$
$10^0=\ \ 1$	$10^{-0}=\frac{1}{10^0}=1$

$$\frac{10^2\times10^3\times10^7}{10^5\times10\times10^8}=\frac{10^{2+3+7}}{10^{5+1+8}}=\frac{10^{12}}{10^{14}}=10^{12-14}=10^{-2}=0.01$$

$$300\times0.00015=3\times10^2\times1.5\times10^{-4}=4.5\times10^{-2}=0.045$$

PROPORTION

We may express the fact that y is proportional to x in the following ways:



1. $y \propto x$.

2. If y is doubled then x is doubled, etc.

3. $y =$ constant times x or $y = kx$.

4. $\dfrac{y_1}{y_2} = \dfrac{x_1}{x_2}$.

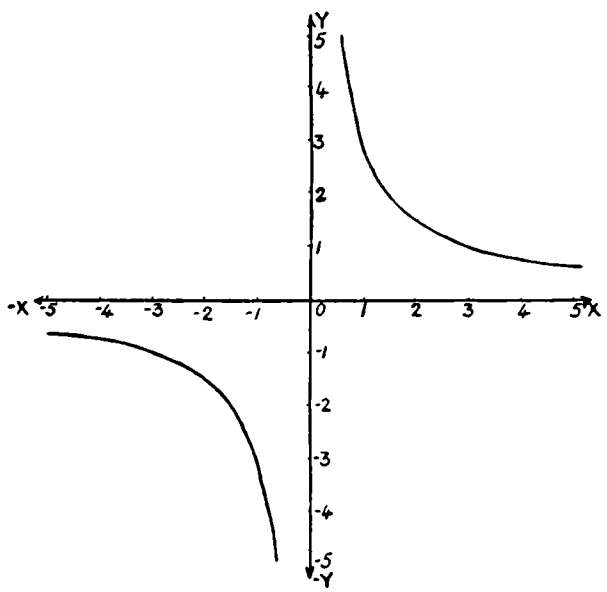
We may express the fact that y is inversely proportional to x in the following ways:

1. $y \propto \dfrac{1}{x}$.

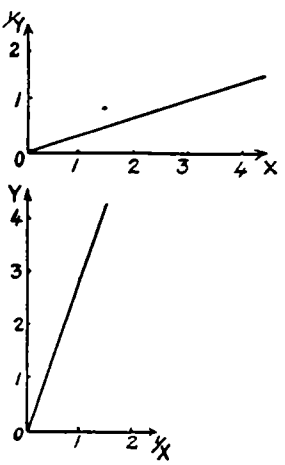
2. If y is doubled then x is halved, etc.

3. $y = \dfrac{\text{constant}}{x}$, or $y = \dfrac{k}{x}$, or $xy = k$.

4. $\dfrac{y_1}{y_2} = \dfrac{x_2}{x_1}$



Graph of $xy = 3$ or $y = \dfrac{3}{x}$
y against x



Graph of $xy = 3$ or $y = \dfrac{3}{x}$
against $\dfrac{1}{y}$ and y against $\dfrac{1}{x}$

Draw a graph of y against x for the case of $xy = -3$.
Draw a graph of y against $1/x$ for the case of $xy = -3$.
Draw a graph of $1/y$ against x for the case of $xy = -3$.
(The first graph gives a rectangular hyperbola, while the latter two give a straight line.)

ANSWERS TO PROBLEMS

CHAPTER 1

1. (*a*) 1.29 sec.; (*b*) 8 min. 14.5 sec.; (*c*) 2.64×10^{13} mi. **2.** 1,020 ft. per min. or 17 ft. per sec. **3.** 35 hr. 12 min. **4.** (*a*) 50 MPH; (*b*) no; (*c*) no. **5.** 4.8 sec. **6.** (*a*) 500 cm. per sec. per sec.; (*b*) 250 cm. per sec.; (*c*) 250 cm. **7.** (*a*) 7.5 m. per sec.; (*b*) 3.75 m. per sec. per sec.; (*c*) 11.1 m. per sec. **8.** (*a*) 150 cm. per sec.^{2}; (*b*) 650 cm. per sec.; (*c*) 1,300 cm. **9.** (*a*) 10 MPH per sec., i.e., 14.7 ft. per sec.^{2}; (*b*) -44 ft. per sec.^{2} **10.** (*a*) 50 ft. per sec.; (*b*) 6 sec.; (*c*) 16.7 ft. per sec.^{2}; (*d*) 30,000 ft.; (*e*) -25 ft. per sec.^{2}; (*f*) 200 ft. **12.** 5 sec. **13.** 5.36 sec. **14.** (*a*) 100 cm. per sec.; (*b*) 200 cm. per sec.; (*c*) 50 cm. per sec.; (*d*) 50 cm.; (*e*) 100 cm. per sec.; (*f*) 200 cm.; (*g*) 150 cm. per sec.; (*h*) 150 cm. **15.** 3,000,000, i.e., 3×10^6 cm. per sec.^{2} **16.** (*a*) 30.6 sec.; (*b*) 4.6×10^5 cm.; (*c*) 61.2 sec. **17.** (*a*) 24 MPH; (*b*) 36 MPH; (*c*) 36 MPH; (*d*) 48 MPH; (*e*) 24 MPH. **18.** (*a*) 16 ft., 64 ft., 144 ft., 256 ft.; (*b*) 48 ft., 80 ft.; (*c*) 1, 3, 5; (*d*) 17×16, i.e., 272 ft.

CHAPTER 2

1. (*a*) 154 lb.; (*b*) 26.7 ft. per sec.; (*c*) 18.2 MPH. **2.** (*a*) 980,000 dynes; (*b*) 980,000 dynes. **3.** (*a*) Yes; (*b*) zero. **4.** (*a*) No; (*b*) yes. **5.** (*a*) Zero; (*b*) 0.5 gm. wt., i.e., 490 dynes. **6.** (*a*) Zero; (*b*) zero; (*c*) 4,900 dynes. **7.** (*a*) 2,000 cm. per sec.^{2}; (*b*) 60,000 dynes. **8.** (*a*) 150 cm. per sec.^{2}; (*b*) 22.5×10^7 dynes. **9.** (*a*) 300 cm. per sec.^{2}; (*b*) 4.5×10^8 dynes. **10.** (*a*) Zero; (*b*) zero; (*c*) 100 kg. wt., i.e., 9.80×10^7 dynes. **11.** (*a*) 5,000 cm.; (*b*) 400 cm. per sec.^{2}; (*c*) 2×10^8 dynes, i.e., 2.04×10^5 gm. wt.; (*d*) 254 kg. wt.; (*e*) zero; (*f*) 50 kg. wt.; (*g*) 3.3×10^8 dynes. **12.** (*a*) 8,000 cm.; (*b*) 2.8×10^6, i.e., 2,800,000 dynes (air resistance and some friction with the ice). **13.** (*a*) 15×10^5 cm. per sec.^{2}; (*b*) 7.5×10^6 dynes. **14.** (*a*) 2.25×10^7 cm. per sec.^{2}; (*b*) 11.25×10^7 dynes. **15.** The person falls forward. **16.** (*a*) You fall backward; (*b*) you fall forward. **17.** (*a*) 70 kg. wt.; (*b*) zero; (*c*) less than 70 kg. wt.; (*d*) greater than 70 kg. wt.; (*e*) greater than 70 kg. wt.; (*f*) less than 70 kg. wt. **18.** (*a*) 3.5×10^8 dynes; (*b*) you slide tangentially off the turntable. **19.** (*a*) Uniform velocity; no resultant force; (*b*) acceleration; there is a force increasing velocity; (*c*) retardation; there is a force decreasing velocity; (*d*) acceleration; there is a force which is increasing with time; (*e*) uniform velocity; no resultant force; (*f*) acceleration; there is a force increasing velocity; (*g*) retardation; there is a force decreasing velocity; (*h*) at rest; no resultant force. **20.** 4×10^9 dynes. **21.** Graph A. **22.** (*a*) 75 cm. per sec.; (*b*) $\frac{2}{15}$ sec.;(*c*) 1,125 cm. per sec.^{2}; (*d*) 225,000 dynes; (*e*) 112,500 dynes. **23.** Yes. **24.** 6.67×10^{-6} dynes. **25.** 25 kg. wt. **26.** 6.02×10^{27} gm. **27.** (*a*) 6.67×10^{-7} dynes; (*b*) 6.67×10^{-8} dynes; (*c*) 40.14×10^{19} dynes; (*d*) 980 dynes.

CHAPTER 3

1. 13 steps E.NE. (*a*) 12 steps N.; (*b*) yes. **2.** SW. **3.** Vector sum of velocity of star and velocity of earth. **4.** 102 MPH, approximately W. by SW. **5.** Fore of the mast. **6.** (*a* and *b*) 865 gm. wt.; (*c*) yes, to overcome the inertia of the sled, i.e., to produce the acceleration. **7.** (*a*) 16 ft.; (*b*) 800 ft. **8.** Yes.

9. 3,520 ft. **10.** (*a*) 49,000 cm. per sec., 49,000 cm. per sec.; (*b*) 50 sec.; (*c*) 100 sec.; (*d*) 4,900,000 cm. **11.** 40 ft. per sec. **12.** (*a*) 3.2 ft. per sec.^{2}; (*b*) 40 sec.; (*c*) 128 ft. per sec.; (*d*) 128 ft. per sec. **14.** (*a*) 97 kg.; (*b*) 25 kg. **15.** Yes. **16.** (*a*) 20 MPH backward; (*b*) at rest. **17.** (*a*) slightly in front; (*b*) in a slightly forward direction; (*c*) yes. **18.** 45°. **19.** 300,000 km. per sec., i.e., 3×10^{10} cm. per sec. **20.** (*a*) Yes; (*b*) yes; (*c*) less; (*d*) west; (*e*) east. **21.** (*a*) Down; (*b*) up.

CHAPTER 4

1. The board moves backward; (*a*) yes, you move forward; (*b*) yes, you move forward; (*c*) yes. **2.** Throw away a shoe, etc., etc. (throwing your arms around won't do). **3.** No (if sails bring all the air you set in motion to rest). **4.** (*a*) 10 cm. per sec.; (*b*) 10^6 dynes. **5.** 20 cm. per sec. **6.** (*a*) 6×10^5 kg.$\times$cm. per sec.; (*b*) 6×10^5 kg.$\times$cm. per sec.; (*c*) 200 cm. per sec. in the direction in which the 2,000-kg. boat is moving. **7.** (*a*) 10.5×10^5 gm. cm. per sec.; (*b*) 10.5×10^5 gm. cm. per sec.; (*c*) 5.25×10^7 dynes; (*d*) 5.25×10^7 dynes. **8.** Yes. **9.** (*a*) 10,000 gm. wt.; (*b*) 98 cm. per sec.^{2}; (*c*) 65.3 cm. per sec.^{2}; (*d*) 196 cm. per sec.; 1.96×10^7 gm. cm. per sec.; (*e*) 130.6 cm. per sec.; 1.96×10^7 gm. cm. per sec. **10.** (*a*) 16.64 cm. per sec.; (*b*) 50,000 gm. cm. per sec.; (*c*) 50,000 gm. cm. per sec. **11.** (*a*) 10,000 cm. per sec. in opposite direction; (*b*) 100,000 gm. cm. per sec.; (*c*) $-100{,}000$ gm. cm. per sec. **12.** The 10-gm. ball has a velocity of 10 cm. per sec. in a direction opposite to that in which it struck. The 20-gm. ball has a velocity of 10 cm. per sec. in the same direction as the 10-gm. ball was moving before striking. **13.** Yes. **14.** (*a*) Yes; (*b*) no; (*c*) yes.

CHAPTER 5

1. Zero. **2.** (*a*) 1.96×10^8 ergs; (*b*) 1.96×10^8 ergs; (*c*) 1.96×10^8 ergs. **3.** (*a*) 82.3×10^9 ergs; (*b*) 82.3×10^9 ergs; (*c*) 411.6 watts. **4.** Friction and air resistance. **5.** Friction, air resistance, and inertia of car. **6.** Friction, air resistance, inertia of car, and gravity. **7.** 9.8×10^8 ergs. **8.** 16.95×10^8 ergs. **9.** (*a*) 10^9 ergs; (*b*) 10^9 ergs; (*c*) 40 cm. per sec.; (*d*) 2×10^6 ergs; (*e*) 7.5×10^8 ergs; (*f*) 7.5×10^8 ergs; (*g*) 3.75×10^8 dynes. **10.** (*a*) 4×10^9 ergs; (*b*) 4×10^9 ergs; (*c*) 8×10^6 dynes. **11.** (*a*) 7.8×10^{10} ergs; (*b*) 7.8×10^{10} ergs; (*c*) 156.8 watts. **12.** (*a*) 1.96×10^{11} ergs; (*b*) 1.96×10^{11} ergs; (*c*) 1.96×10^8 dynes; (*d*) 2.5×10^{11} ergs; (*e*) 1.96×10^{11} ergs. **13.** (*a*) 2.2×10^8 ergs; (*b*) 2.2×10^8 ergs. **14.** (*a*) 1.25×10^9 ergs; (*b*) 2.5×10^9 dynes; (*c*) 29.8 cal. **15.** (*a*) 10^6 dynes; (*b*) 15×10^8 ergs; (*c*) 10.5×10^8 ergs; (*d*) 25 cal.; (*e*) 4.5×10^8 ergs; (*f*) 300 cm. per sec. **16.** (*a*) 2.45×10^{12} ergs; (*b*) 4.1×10^{12} ergs. **17.** (*a*) Zero and 30 cm. per sec. (the balls exchange velocities); (*b*) 22.5×10^3 ergs, both before and after; (*c*) 22.5×10^3 ergs before; 11.25×10^3 ergs after impact. **18.** (*a*) 0.1 sec.; (*b*) 3×10^5 dynes; (*c*) 3×10^6 ergs; (*d*) 3×10^6 ergs; (*e*) yes; (*f*) 6×10^5 dynes. **19.** (*a*) 746,000 watts; (*b*) 3.81×10^3 cm., i.e., 38.5 m. **20.** (*a*) At the blade of the oar; (*b*) handle and oarlock; (*c*) 3.3×10^4 gm. wt.; (*d*) $\frac{1}{3}$. **21.** (*a*) 11×10^4 gm. wt.; (*b*) 11. **22.** (*a*) 2.94×10^9 ergs; (*b*) 768 cm. per sec. **23.** (*a*) 7,350 watts, i.e., 9.85 h-p.; (*b*) 2.46 h-p. **24.** (*a*) Yes; (*b*) it has been used up in doing work against the friction of the banks, bottom, and the water itself—the energy has gone into heat. **25.** (*a*) 15 cm. per sec.; (*b*) 22.5 cm., 22.5×10^6 ergs; (*c*) 22.5×10^6 ergs; (*d*) 15 cm. per sec. for 2×10^5 gm. boat and 7.5 cm. per sec. for 4×10^5 gm. boat; (*e*) 33.75×10^6 ergs; (*f*) yes; (*g*) yes. **26.** (*a*) 6,000 cm. per sec.; (*b*) 9×10^7 ergs; (*c*) 4.5×10^4 ergs; (*d*) 2.14 cal. **27.** (*a*) 4.9×10^6 ergs; (*b*) 4.9×10^6 ergs; (*c*) 0.11 cal.; (*d*) 4.45×10^7 ergs per cal.; (*e*) in steady stream. **28.** (*a*) 1.96×10^6 ergs; (*b*) 1.96×10^6 ergs; (*c*) 4.9×10^7 ergs per cal.; (*d*) into warming cardboard tube, thermometer, cork, etc. **29.** (*a*) 14.7×10^{11} ergs; (*b*) 30 m.; (*c*) 27 m. **30.** (*a*) 3.125×10^{12} ergs; (*b*) 3.125×10^8 dynes, 312.5 cm. per sec.^{2}; (*c*) 3,185 cm. **31.** 5×10^7 dynes. **32.** (*a*) 25 cm. per sec.; 50 cm. per sec.; (*b*) 12.25×10^5 ergs; (*c*) 12.25×10^5 ergs; (*d*) 970 gm. **33.** The lever arm is greater. **34.** (*a*) The earth; (*b*) yes; (*c*) heat.

CHAPTER 6

1. (*a*) *B*, *C*, *A*; (*b*) *B*, *C*, *A*. **2.** Lower. **3.** (*a*) Yes; (*b*) yes; (*c*) yes. **4.** By the vertical height. **5.** The volume is halved. **6.** 875 cc. **7.** Yes. **8.** (*a*) 15.6×10^7 dynes; (*b*) 7.8×10^7 dynes; (*c*) 15.6×10^7 dynes. **9.** (*a*) 14×10^8; (*b*) yes. **10.** 0.88. **11.** (*a*) Yes; (*b*) 627 cc.; (*c*) 2,507 gm. of silver. **12.** 166,667 gm. of wood. **13.** Less; buoyancy of air increased. **14.** 1,034 cm., i.e., 34 ft. **15.** 0.148 cc. **16.** The air inside you exerts an outward pressure. **17.** 0.015 gm. wt. **18.** (*a*) pressure 5 cm. of mercury, volume 30 cc.; (*b*) 2 cc.; (*c*) 0.0024 gm. wt. **19.** (*a*) 150 cc.; (*b*) 76 cc. **20.** 1.09×10^{11} dynes, i.e., 1.11×10^8 gm. wt. **21.** (*a*) 27.4×10^5 gm. wt.; (*b*) 1.8×10^5 gm. wt.; (*c*) 25.6×10^5 gm. wt. **22.** (*a*) 9×10^4 gm. wt. on sides and 2.25×10^4 gm. wt. on ends; (*b*) 45 kg.; (*c*) 0.9 gm. per cc.

CHAPTER 7

1. Ice floats in water, frozen pipes often burst, etc. **2.** It is less dense than the surrounding air. **3.** 4.3 atmospheres. **4.** (*a*) 0.0000898, i.e., 8.98×10^{-5} gm. per cc.; (*b*) 1,055 cc.; (*c*) 0.000085 gm. per cc. **5.** (*a*) 2 gm.; (*b*) 22.4 liters; (*c*) 8.31×10^{7} dynes×cm. per °C.; (*d*) 8.31×10^{7} dynes×cm. per °C. **6.** (*a*) Yes; (*b*) no; (*c*) 74.8 cm. **7.** 3.6 mm. **8.** 45 cm. **9.** 12 sq. cm. **10.** 1,801 cc. **11.** Soda glass. **12.** Less than 7.8 gm. per cc. **13.** (*a*) Mercury; (*b*) alcohol; (*c*)

	Alcohol	Mercury
Boiling point	172.4° F.	675° F.
Freezing point	−202° F.	− 38° F.

(*d*) −40° F. is also −40° C. **14.** 16.7×10^{5} gm. **15.** (*a*) fall; (*b*) liquid rises. **16.** (*a*) Rises to top of tube, i.e., 100 cm.; (*b*) 60 cm. up tube. **17.** (*a*) Yes; (*b*) no; (*c*) no; (*d*) yes; (*e*) the whole mass of water must be cooled to 4° C. before any freezing can start—this takes longer for a large mass of water than for a smaller one. **18.** Less. **19.** 1,820 cc. **20.** (*a*) 2 gm.; (*b*) 101.3 cm. of mercury or 4/3 atmospheric. **21.** (*a*) 777.7 cc.; (*b*) 333.3 cc.

CHAPTER 8

1. (*a*) 36 cal.; (*b*) 0.5 cm. **2.** (*a*) Through the plaster; (*b*) cooler; (*c*) cooler (see problem 18, chap. ix); (*d*) between the laths. **3.** (*a*) 115.2 gm. of ice; (*b*) 9.4° C. **4.** (*a*) 270,000 cal.; (*b*) 11.35×10^{12} ergs. **5.** 190,000 cal. **6.** 0.88. **7.** (*a*) 74.5 cal.; (*b*) 3.2×10^{9} ergs. **8.** 0.545. **9.** (*a*) 4×10^{4} cal.; (*b*) 8.64×10^{4} gm.; (*c*) 1.93° C.; (*d*) convection.

CHAPTER 9

1. Twice. **2.** (*a*) 19.8×10^{7} gm. wt.; (*b*) 21.6×10^{7} gm. wt.; (*c*) helium is non-inflammable. **3.** 3.56×10^{11} molecules. **4.** (*a*) 6.06×10^{23} molecules; (*b*) 22.4 liters; (*c*) 22.1 liters. **5.** Since temperature of the two gases is the same, $\frac{1}{2}m_1v_1^2$ will be equal to $\frac{1}{2}m_2v_2^2$. Hence the pressure of each gas is proportional to the number of molecules of that gas present. Hence the total pressure is proportional to the total number of molecules present, i.e., the total pressure is the sum of the pressures exerted by each gas. **6.** (*a*) Yes; (*b*) no. **7.** 33.9×10^{9} (cm. per sec.)^2 for hydrogen molecule; 2.42×10^{9} (cm. per sec.)^2 for nitrogen molecule. **8.** It contains mercury molecules, the number and pressure of which increase with temperature. **9.** Lower temperature than 100° C. **10.** 70° C. **11.** 85° C. **12.** (*a*) 25° C.; (*b*) 55 per cent. **13.** Yes, the temperature of the water and the temperature at which dew is formed would be more nearly equal because silver is a much better conductor of heat. **14.** You can get rid of heat by perspiration on a dry day. **15.** The breeze helps evaporation to take place and hence has a cooling effect. **16.** Yes. **17.** No. **18.** The warmer air below has a greater *KE* than the cooler air above, hence the dust particles receive more momentum from below than above and they rise.

CHAPTER 10

1. (*a*) 32 dynes; (*b*) 4 dynes; (*c*) 16 dynes; (*d*) $\frac{4}{81}$ dynes. **2.** (*a*) 0.05 e.s.u. of field strength; (*b*) *A* to *B*; (*c*) 1 dyne; direction, *B* to *A*; (*d*) 1 dyne; direction, *A* to *B*. **3.** The negative charge on the pen induces + and − charges on the paper. The − charge on the pen attracts the + charge on the paper (the − charge on the paper is farther away). **4.** No; any charge produced by the rubbing will immediately be conducted to ground through your body. **5.** (*a*) Negative; (*b*) more; (*c*) yes; (*d*) positive; (*e*) yes. **6.** (*a*) No; (*b*) when *BC* is charged by induction, it acquires a charge opposite in sign to that of the inducing charged rod *A*—these charges attract one another, so more energy is used up in drawing *A* away from *BC* than when *BC* is neutral. **7.** S. pole of a magnet. **8.** (*a*) 0.2 gauss; (*b*) 0.346 gauss. **9.** (*a*) No; (*b*) 0.9 dynes; (*c*) zero; (*d*) 9.0 dynes×cm. **10.** Yes; it helps the molecular magnets to orient themselves. **11.** (*a*) 0.05 gauss; (*b*) 0.05 dynes; (*c*) 1 dyne. **12.** $\frac{1}{16}$ gauss. **13.** (*a*) 2 volts per cm.; (*b*) 2×10^{7} dynes; (*c*) 10 joules; (*d*) 10 joules; (*e*) 1 volt per cm. **14.** (*a*) 12 ohms; (*b*) 0.75 watts; (*c*) 450 joules. **15.** 200 coulombs. **16.** (*a*) 5 amp.; (*b*) 66,000 joules; (*c*) 0.18 cents; (*d*) 3.6×10^{6}. **17.** (*a*) 10 amp.; (*b*) 2,000 watts; (*c*) 100 amp.; (*d*) 200,000 watts. **18.** (*a*) 20 ohms; (*b*) 1.25 watts; (*c*) 45,000 ohms; (*d*) 0.18 watts;

(*e*) 14.31×10^{-18} joules. **19.** (*a*) 20 cents; (*b*) 2.27 amp.; (*c*) parallel; (*d*) no; (*e*) 4.54 amps. **20.** (*a*) 80,000 cal.; (*b*) 33.6×10^{4} joules; (*c*) 5.5 amp.; (*d*) 9 min. 15 sec.; (*e*) 0.9 cents. **21.** (*a*) 62.8 gauss; (*b*) 31,400 gauss; (*c*) 628,000 lines of force; (*d*) 0.628 volts; (*e*) 0.314 amp.; (*f*) 0.00314 coulombs; (*g*) 62.8 volts. **22.** Yes. **23.** (*a*) Yes; (*b*) yes. **24.** 20 dynes. **25.** No. **26.** (*a*) In series; (*b*) 14; (*c*) about 2 watts; (*d*) 32 ohms; (*e*) it would burn out immediately.

CHAPTER 11

1. 1.008 gm. of hydrogen, 8 gm. of oxygen. **2.** (*a*) 965 coulombs; (*b*) 0.01 gm. of hydrogen ; 0.08 gm. of oxygen; (*c*) 3.03×10^{21} molecules of hydrogen; 1.51×10^{21} molecules of oxygen; (*d*) +1, −2 (the charge on the electron is taken as unity). **3.** (*a*) 48,250 coulombs; (*b*) 17.73 gm. of chlorine; (*c*) 5.6 liters; (*d*) 3.03×10^{23}; (*e*) 3.03×10^{23}; (*f*) cathode. **4.** (*a*) +2; (*b*) −1; (*c*) +3. **5.** (*a*) 12.15 gm.; (*b*) 79.9 gm.; (*c*) 9.0 gm. **6.** (*a*) 3.03×10^{23}; (*b*) 6.06×10^{23}; (*c*) 2.02×10^{23}. **7.** (*a*) Cathode; (*b*) 0.21 gm.; (*c*) 187.8 coulombs; (*d*) 1,878 sec. **8.** (*a*) 1.59×10^{-15} joules; (*b*) yes. **9.** 1.59×10^{-15} joules, i.e., 1.59×10^{-8} ergs. **10.** 16.65×10^{-25} gm. **11.** 1.37×10^{8} cm. per sec. **12.** (*a*) 1.59×10^{-15} joules, i.e., 1.59×10^{-8} ergs; (*b*) $\frac{1.37}{4.7}\times10^{8}$, i.e., 0.29×10^{8} cm. per sec. **13.** 4.6×10^{-12} dynes. **14.** 90 atoms of mass 20, 10 atoms of mass 22. **15.** a_2^4, an alpha particle. **16.** a_2^4, an alpha particle.

17.

	Th	MsTh1	MsTh2	RaTh	ThX	Th	ThA	ThB	ThC	ThC′	ThD
Atomic number.....	90	88	89	90	88	86	84	82	83	84	82
Atomic weight.....	232	228	228	228	224	220	216	212	212	212	208

18. (*a*) RaTh; (*b*) ThB. **19.** (*a*) Yes; (*b*) yes; (*c*) no.

CHAPTER 12

1. (*a*) 1,250 cm.; (*b*) 1.25 vibrations per sec. **2.** (*a*) 48,000 cm. per sec.; (*b*) 48 cm. **3.** Medium *A*. **4.** (*a*) Longitudinal; (*b*) no; the transverse wave is stopped. **5.** (*a*) The waves traveling along the surface; (*b*) 13.4 sec. **6.** Source of waves is 500 m. from both *A* and *C*. Source is 400 m. from *B* and on a line at right angles to *ABC* through *B*. **7.** (*a*) The nearer the waves come to the shore, the slower they travel; thus the waves in deeper water are given a chance to catch up with them; (*b*) refraction; (*c*) no. **8.** Yes (the water waves can dissipate their energy through eddies). **9.** More. **10.** (*a*) To the right; (*b*) to the left. **11.** (*a*) 4 cm.; (*b*) 200 cm. per sec.

CHAPTER 13

1. (*a*) 165,000 cm., i.e., 1.65×10^{5} cm.; (*b*) 1 mi. (approx.) **2.** 165 m. or 0.1 mi. (approx.). **3.** 1,260 m. per sec., i.e., 1.26×10^{5} cm. per sec. **4.** $\sqrt{\frac{28}{32}}$ i.e. $\sqrt{\frac{7}{8}}$ **5.** $\sqrt{\frac{1.66}{1.41}\times\frac{2}{4}}$. **6.** (*a*) Density is increased in same ratio as pressure; (*b*) Boyle's Law; (*c*) there is no effect; (*d*) yes. **7.** (*a*) $\frac{d_0}{d_{27}}=\frac{273+27}{273}=\frac{300}{273}$; (*b*) Charles's Law; (*c*) $\frac{d_0}{d_t}=\frac{273+t}{273}=\frac{T°\text{ A.}}{273}$; (*d*) $\frac{v_T}{v_0}=\sqrt{\frac{T}{273}}$. **8.** (*a*) In water; (*b*) path *C*. **9.** (*a*) Transverse; (*b*) yes. **10.** (1) *a*; (2) *c*′. **11.** (*a*) 2×10^{4} cm. per sec.; (*b*) 200 cm., 100 cm., 66.7 cm., 50 cm.; (*c*) 100 per sec., 200 per sec., 300 per sec., 400 per sec.; (*d*) yes. **12.** (*a*) 166 vibrations per sec.; (*b*) 332, 498, 664 vibrations per sec.; (*c*) all of the harmonics. **13.** (*a*) 100 cm.; (*b*) no. **14.** (*a*) 359 m. per sec.; (*b*) 1 sec.; (*c*) 1 sec. **15.** 150 per sec. **16.** (*a*) Yes; (*b*) no. **17.** (*a*) 3 ft. per sec.; (*b*) 1 step, $\frac{2}{3}$ step; (*c*) 3 for the lady, 2 for the gentleman; (*d*) they are in step once every 3 sec., i.e., $\frac{1}{3}$ of a time a sec.; yes: $n_1=1$, $n_2=\frac{2}{3}$, $n_1-n_2=\frac{1}{3}$. **18.** (*a*) Yes; (*b*) no; (*c*) 3 per sec. **19.** (*a*) 4 cm.; (*b*) 8,300 per sec.; (*c*) yes. **20.** (*a*) Greater than 1,000 per sec.; (*b*) less than 1,000 per sec. **21.** (*a*) 32,000 cm.; (*b*) 32 cm.; (*c*) 1037.5 per sec.; (*d*) 965

per sec. **22.** (*a*) Greater than 1,500 per sec.; (*b*) less than 1,500 per sec. **23.** (*a*) 3,320 cm.; (*b*) 150 waves; (*c*) 1,650 waves, 1,650 per sec.; (*d*) 1,350 per sec. **24.** (*a*) $\frac{n'}{n}=\frac{V+v}{V}$; (*b*) $\frac{\lambda'}{\lambda}=\frac{V-v}{V}$; (*c*) $\frac{n'}{n}=\frac{V}{V-v}$. **25.** (*a*) Cooler; (*b*) the sound waves remain close to the ground due to refraction (change in velocity due to temperature differences).

CHAPTER 14

1. Amplitude. **2.** (*a*) 90°; (*b*) 20°; (*c*) 110°. **3.** (*a*) −12.5 cm., i.e., 12.5 cm. behind the mirror; (*b*) virtual, erect, and diminished. **4.** (*a*) No; (*b*) no. **5.** (*a*) 150 cm.; (*b*) real. **6.** Yes. **7.** (*a*) −60 cm. (behind the mirror), 18 cm. high; (*b*) virtual, erect, and magnified. **8.** A blur is produced; the image is at infinity. **9.** (*a*) +60 cm. (in front of the mirror), 6 cm. high; (*b*) real inverted, same size as object. **10.** (*a*) +45 cm., 3 cm. high; (*b*) real, inverted, diminished. **12.** (*a*) Concave mirror; (*b*) virtual. **13.** Less. **14.** (*a*) −2.25 cm. (on same side of lens as object), 0.5 cm. high; (*b*) virtual, upright, diminished. **15.** (*a*) No; (*b*) no. **16.** (*a*) 20 cm.; (*b*) no. **17.** (*a*) Convex lens; (*b*) virtual, so that the image is upright and magnified. **18.** (*a*) −15 cm., magnification 2.5; (*b*) virtual, upright, magnified; (*c*) infinity; (*d*) no; (*e*) +20; (*f*) real, inverted, same size as object; (*g*) closer to book. **19.** (*a*) Convex; (*b*) the focal length of the lens away from the lens; (*c*) away from film. **20.** (*a*) Shorter focal length; (*b*) in front of retina; (*c*) concave lenses; (*d*) behind the retina; (*e*) convex lenses. **21.** (*a*) His feet, being nearer to the camera than his head, are magnified more than the rest of his body; (*b*) yes. **22.** (*a*) 50 cm. from the objective lens; (*b*) 2 cm. high; (*c*) a virtual image 30 cm. high and 28 cm. from the eyepiece. **23.** Diffraction. **24.** (*a*) Shorter; (*b*) more blue; (*c*) more red. **25.** To the blue. **26.** (*a*) 4.55×10^{14} vibrations per sec., 7.5×10^{14} per sec.; (*b*) blue; (*c*) wave-lengths changed. **27.** Less deep. **28.** (*a*) *A*; (*b*) *A*; (*c*) red. **29.** (*a*) 2,000 A or 2×10^{-5} cm.; (*b*) 3.2×10^{-5}; (*c*) 0.063 cm., 0.08 cm. **30.** (*a*) Away from earth; (*b*) 5×10^{6} cm. per sec.

CHAPTER 15

1. 6,170° A. or 5,897° C. **2.** (*a*) 10,720° A. or 1.07×10^{-4} cm.; (*b*) 16,100° A. or 1.61×10^{-4} cm. **3.** $\frac{81}{16}=5.06$. **4.** 16. **5.** (*a*) Yes; (*b*) bright line spectrum (in this case it is called the flash spectrum, since it flashes out immediately the sun's disk is covered). **6.** (*a*) 19.65×10^{-9} ergs; (*b*) 3.9×10^{-12} ergs; (*c*) 19.65×10^{-7} ergs. **7.** (*a*) The sun; (*b*) it is absorbed and warms the soil; (*c*) much longer; (*d*) no. **8.** Yes. **9.** Less. **10.** Yes. **11.** No. **12.** (*a*) Less; (*b*) $\frac{1}{1,846}$. **13.** 7.3×10^{-28} gm. **14.** (*a*) 1.03×10^{9} cm. per sec.; (*b*) 0.71 Å or 7.1×10^{-9} cm. **15.** 6.55×10^{-30} cm. **16.** (*a*) 4.5×10^{20} ergs or 4.5×10^{13} joules; (*b*) 1.25×10^{6} dollars, i.e., $1,250,000; (*c*) ?.

INDEX

A

Aberration, 36, 168
Acceleration, 5, 15, 16; of gravity, 7
Alpha particle, 130, 137, 138
Ampere, 115
Amplitude, 145
Anderson, 140
Angstrom unit, 177
Anode, 132
Appendix, 194
Archimedes' principle, 74
Atmospheric pressure, 68
Atom, 99
Atomic number, 130
Avogadro: number, 100; hypothesis of, 100, 103

B

Balmer, 187
Barometer, 71
Beats, 159
Beta particle, 138
Bohr, 129, 186
Boyle's Law, 71

C

Calorie, 91
Calorimeter, 91
Cathode, 132
Cathode rays, 135
Charge, electrostatic unit of, 109
Charles's Law, 83
Circuits: in series, 116; in parallel, 117
Collision: elastic, 41; inelastic, 41
Compression, 144, 156
Compton, A. H., 189
Conduction, 94
Conductivity, thermal, 95
Conductors, electrical, 109
Conservation: of momentum, 40; of mechanical energy, 53; of energy, 56
Convection, 94
Coolidge, 135
Corona, light, 175
Coulomb of charge, 114
Coulomb law of force, 108
Crests, wave-motion, 143
Current, alternating, 123

D

Davisson, 191
De Broglie, 191
Density, 68
Dielectric constant, 108
Diffraction: of waves, 151; of sound, 157; of light, 175; grating, 176; of X-rays, 190; of electrons, 191
Dirac, 191
Disintegration, artificial, 139
Dispersion, of light, 171
Doppler effect, 152
Dyne, 14

E

Earth, diameter of, 24
Einstein, 185, 189
Electric currents, 115
Electric fields, work against, 113
Electric intensity, 110
Electrical circuits: in parallel, 117; in series, 117
Electrolysis, 132
Electromagnet, 120

Electromagnetic radiation, 182, 185
Electrons: unit of negative charge, 109; arrangement in atom, 129; nuclear, 130; planetary, 130; mass of, 135; diffraction of, 191
Energy: definition of, 52; kinetic, 52, potential, 53, conservation of, 56
Erg, 47
Expansion: of solids, 81; of gases, 82; of liquids, 82
Eye, the, 179

F

Faraday: electromagnetic induction, 122; Laws of Electrolysis, 133
Fizeau, 166
Fluids, 67
Force: centripetal, 14; unit of, 14; resolution of, 27; moment of, 57
Foucault, 166
Fraunhofer, 176, 192
Frequency of vibration, 145
Fresnel, 175

G

Galileo, 15, 191
Gamma ray, 138
Gases, kinetic theory of, 101
Gauss, 111
Germer, 191
Gramaldi, 175
Gravitation, Law of, 17
Gravity, center of, 58

H

Heat: of fusion, 92; of evaporation, 93
Helium atom, 130
Humidity, 104
Hydrogen atom, 130

I

Ions, 133, 134
Inclined plane, 28
Induction: electrostatic, 109· electromagnetic, 121
Inertia, 13
Insulator, 109
Interference, of light, 174
Isotope, 137, 139

J

Joule, 47

K

Kepler, 17
Kinetic energy, 52
Kinetic theory, of gases, 101

L

Latent heat: of fusion, 92; of evaporation, 93
Laue, 190
Lemon, 102, 130
Lenses, 171
Levers, 56
Light: velocity of, 36, 138, 166; wave nature of, 166; reflection of, 167; wave-length of, 167; aberration of, 168; refraction of, 170; dispersion of, 171; interference of, 174; diffraction of, 175; quanta, 185
Loops, 150
Lyman, 187

M

Magnetic: field strength, 111; pole, 111; field, 120
Magnetism, 110
Maxwell, 102, 166, 186
Mechanical advantage, 57
Mechanical equivalent, of heat, 58
Michelson, 166
Mirrors, 167
Molecular weight, 100
Molecule, 99
Moment of force, 57
Momentum: defined, 39; Law of Conservation of, 40

N

Neutron, 139
Newcomb, 166
Newton: first two laws, 14, 43; Law of Gravitation, 17; third law, 37; rings, 174, 181
Nodes, 150, 158
Nucleus, 129

O

Ohm's Law, 116
Overtones, 159

P

Pendray, 174
Period, of a wave, 145
Periodic Table, 131
Permeability, 120
Phase, 145
Photoelectron, 135, 185, 189
Photons: nature of, 185; of X-radiation, 189; mass of, 190; momentum of, 190
Pitch, 156
Planck's constant, 185
Polarization: of waves, 153; of light waves, 176
Positive ions, 137
Positron, 140
Potential difference, 114
Potential energy, 52
Pressure: atmospheric, 68; definition of, 68
Projectiles, 30 ff
Proportion, 196
Proton, 130, 139

Q

Quality, of sound, 159

R

Radiation: loss of heat by, 94; black-body, 183
Radioactivity, 138
Rarefaction, 144, 156
Ray, 147
Reflection, 147, 167
Refraction, 148, 170
Refractive index, 170
Resistance, specific: defined, 119; table of, 119
Resonance, 157
Roemer, 166
Rutherford, Lord, 129, 139

S

Scalars, 25
Schrödinger, 191
Sound: wave nature of, 156; quality of, 159
Specific heat, 91
Specific inductive capacity, 108
Spectroscope, 174
Spectrum: of light, 174; continuous, 183; atomic, 184; bright-line, 184; dark-line, 184; of hydrogen atom, 186
Speed: instantaneous, 2; uniform, 2; average, 3, 5
Standard conditions, of temperature and pressure, 86
Stationary states, of atom, 186
Stefan's Law, 183

T

Telescope, 180
Temperature: defined, 80; scales, 81
Thomson, G. P., 191
Thomson, Sir J. J., 137
Troughs, in waves, 143
Tyndall, 183

U

Units: of length, 1; of time, 1; of force, 14; of energy, 47; of work, 47; of electric charge, 109, 114; of magnetic field, 111; magnetic pole, 111

V

Valence, 134
Vapor pressure: in light of kinetic theory, 103; curve for water, 104
Vectors, 15, 25
Velocity: a vector quantity, 14; terminal, 17; of light, 36, 138, 166; of a wave, 146; of sound, 156
Volt, 114

W

Watt, 47, 115
Wave: motion, 143; length, 145; fronts, 147; mechanics, 191
Waves: transverse, 144; longitudinal, 144, 146; water, 144; period of, 145; velocity of, 146; reflection of, 147; refraction of, 148; standing, 149; diffraction of, 151; particle characteristics of, 189; electron, 191
Weight, 16
Wien's Law, 183
Work, 47

X

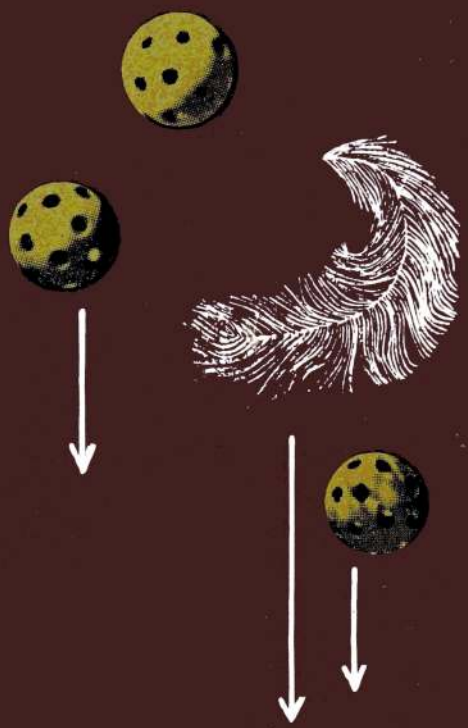
X-rays: from cathode rays, 135; characteristic and continuous, 188; diffraction of, 190

Y

Young, 175

PRINTED IN U.S.A.

Cover by DOUGLAS YOUNGQUIST



exploring in physics

This volume, one of the *New Plan Texts* of the University of Chicago, was prepared as a companion to Harvey B. Lemon's introductory text for physics students, *From Galileo to the Nuclear Age*. Whereas the latter presents the fundamentals of physics in a general way, *Exploring in Physics* focuses attention on specific problems, showing the practical application of the fundamental principles. Each chapter discusses the physical laws used in the solution of the problems while examples and questions indicate methods of approach.

REGINALD J. STEPHENSON *is professor of physics, College of Wooster.*

PHOENIX SCIENCE SERIES
published by The University of Chicago Press